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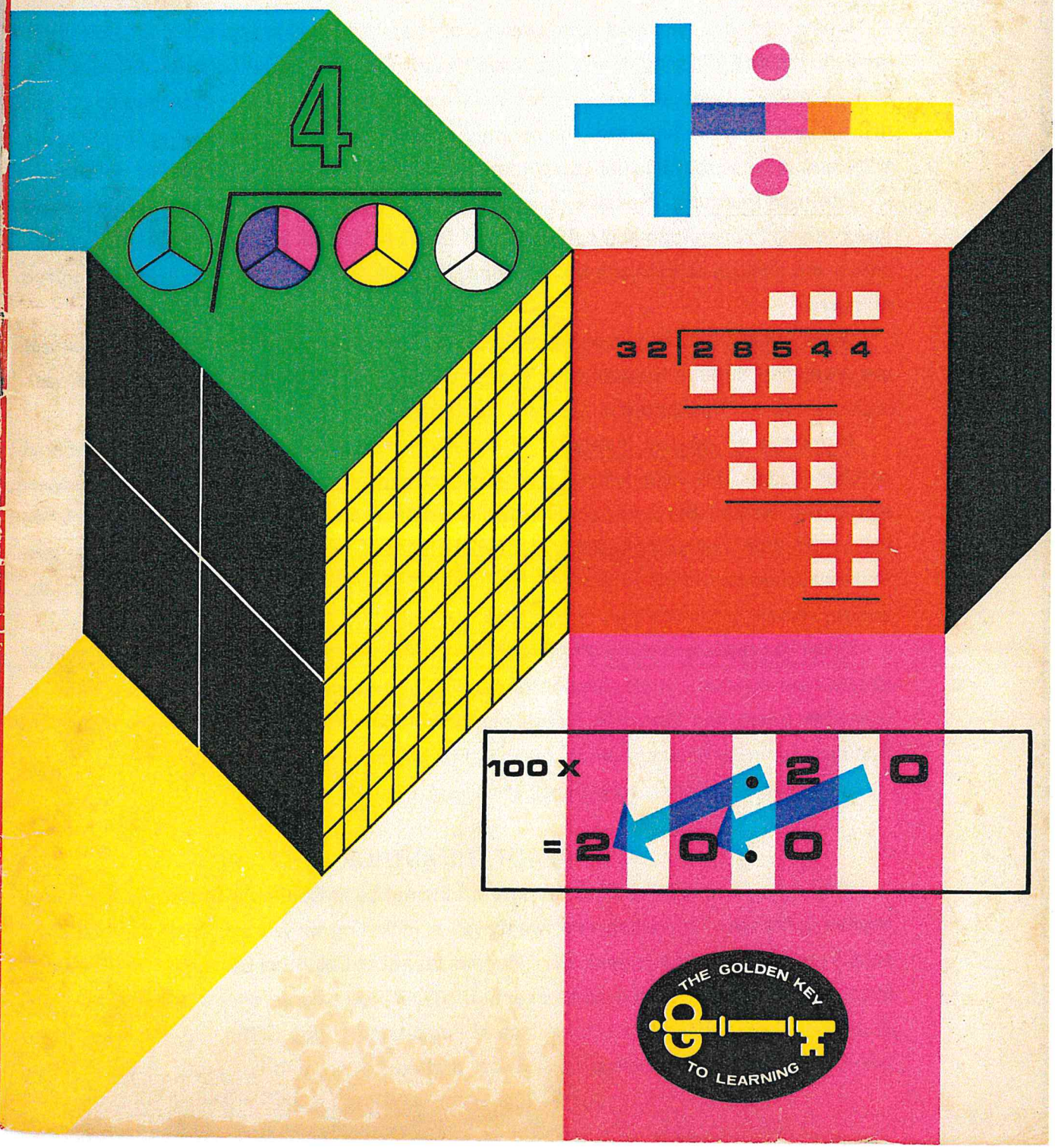
MATHEMATICS—GRADE 6

Mastering Common Fractions and Decimal Fractions

A Workbook with Self-Teaching Text and Learning Exercises

by IRVING ADLER, Ph.D.



TO THE PARENT AND TEACHER

MATHEMATICS—GRADE 6 is based on a careful study of the methods used to teach mathematics both in this country and abroad. The aim is to develop understanding as a foundation for skill. This book includes the best practices found anywhere and has the freshness and originality that are characteristic of all the author's books.

The contents of this volume are conveniently summarized under five headings: whole numbers, common fractions, decimal fractions, properties of rational numbers, and other concepts and skills.

Whole numbers. Numbers up to twelve places are introduced. Addition and subtraction are extended to 6-place numbers. The meanings of multiplication and division are reviewed. Multiplication is extended to the use of 3-place multipliers. Division is extended to 2-place divisors and 6-place dividends. The pyramid form of long division is used at first. Then a transition is made to the usual non-pyramid form of long division, and to short division.

Common fractions. The book reviews the meaning of fraction, changing to higher and lower terms, and addition and subtraction of fractions and mixed numbers. Then multiplication and division of fractions are developed through multiplication and division of mixed numbers.

Decimal fractions. The meaning of decimals, and addition and subtraction of decimals are developed through thousandths. The first steps in multiplication and division of decimals are taken. The concept of per cent is introduced.

Properties of rational numbers. To develop a deeper understanding of the rational number system, this volume formulates and uses explicitly the following structure properties of the system: the commutative laws of addition and multiplication, the associative laws of addition and multiplication, the distributive law, and the law of one. They are used in the development of multiplication and division of fractions and the division of decimals, and for reviewing the addition of fractions.

Other concepts and skills. Perimeter, area, rate, average, addition and subtraction of denominate numbers, multiplication of denominate numbers by whole numbers, Roman numerals above one thousand, prime numbers, prime factors of a whole number, and finding a common denominator by means of prime factors.

Problems and test exercises are provided at frequent intervals.

ABOUT THE AUTHOR

Irving Adler received his Ph.D. in pure mathematics at Columbia University. He was a teacher in the New York schools for twenty years, including six years as chairman of a high-school mathematics department. Dr. Adler has taught at Columbia University and Bennington College, and was the keynote speaker at the Washington conference of state supervisors of mathematics, called by the U.S. Office of Education in June, 1961. He is the author of about fifty books on mathematics, science, and education, and co-author of thirty more.

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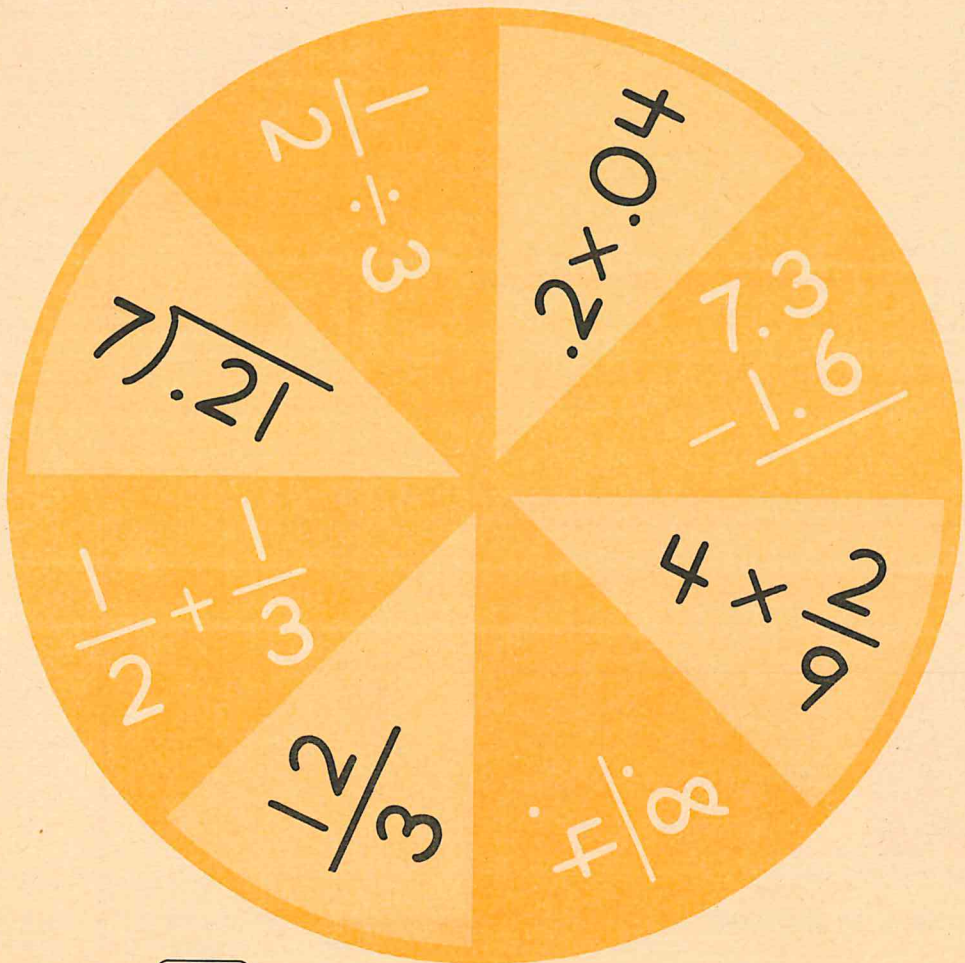
by IRVING ADLER, Ph.D.

Author of *The Giant Golden Book of Mathematics*, *The New Mathematics*, and *Magic House of Numbers*

Illustrated by Mac Shepard



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Three-Digit Numerals

Hundreds	Tens	Ones	
5	1	6	= 5 hundreds + 1 ten + 6 ones.
2	3	8	= ☐ hundreds + ☐ tens + ☐ ones.
4	9	0	= ☐ hundreds + ☐ tens + ☐ ones.
3	0	5	= ☐ hundreds + ☐ tens + ☐ ones.

In a three-digit numeral, the third digit from the right shows the number of hundreds.

Write each of these numbers in numerals:

four hundred twenty-five =	4	2	5.
six hundred seven =	6	0	7.
one hundred nineteen =			
four hundred eight =			
two hundred eighty =			

Write in the rings as you count

by ones: 597 ◯◯◯◯◯

by fives: 285 ◯◯◯◯◯

by tens: 390 ◯◯◯◯◯

by fifties: 150 ◯◯◯◯◯

by hundreds: 208 ◯◯◯◯◯

1 hundred = 10 tens. 50 tens = 5 hundreds.

2 hundreds = ☐ tens. 60 tens = ☐ hundreds.

3 hundreds = ☐ tens. 70 tens = ☐ hundreds.

4 hundreds = ☐ tens. 80 tens = ☐ hundreds.

Exchange 10 tens for 1 hundred.

3 hundreds + 15 tens = 4 hundreds + 5 tens.

4 hundreds + 11 tens = 5 hundreds + ☐ ten.

7 hundreds + 13 tens = ☐ hundreds + 3 tens.

6 hundreds + 10 tens = ☐ hundreds + 0 tens.

5 hundreds + 16 tens = ☐ hundreds + ☐ tens.

8 hundreds + 12 tens = ☐ hundreds + ☐ tens.

Exchange 1 hundred for 10 tens.

5 hundreds + 2 tens = 4 hundreds + 12 tens.

6 hundreds + 1 ten = 5 hundreds + ☐ tens.

3 hundreds + 4 tens = ☐ hundreds + 14 tens.

7 hundreds + 0 tens = ☐ hundreds + ☐ tens.

8 hundreds + 3 tens = ☐ hundreds + ☐ tens.

4 hundreds + 5 tens = ☐ hundreds + ☐ tens.

Thinking Out Addition

423 + 4 = 400 + 23+4 = 400 + 27 = 427.

615 + 5 = 600 + 15+5 = 600 + ☐ = ☐.

344 + 5 = 300 + 44+5 = 300 + ☐ = ☐.

512 + 6 = ☐. 721 + 7 = ☐.

275 + 8 = 200 + 75+8 = 200 + 83 = ☐.

318 + 7 = 300 + 18+7 = 300 + ☐ = ☐.

256 + 4 = 200 + 56+4 = 200 + ☐ = ☐.

427 + 9 = ☐. 513 + 8 = ☐.

812+30=800+ 10+30 +2=800+ 40 +2=☐.

627+60=600+ 20+60 +7=600+ ☐ +7=☐.

531+40=500+ 30+40 +1=500+ ☐ +1=☐.

256+20=☐. 671+10=☐.

325+100= 300+100 +25= 400 +25=☐.

516+300= 500+300 +16= ☐ +16=☐.

241+400= 200+400 +41= ☐ +41=☐.

432+300=☐. 641+200=☐.

Add.

2 1 7	6 2 4
+4 3 2	+1 5 5
☐	☐

1	
5 1 6	6+7=13=
+2 2 7	1 ten+3 ones.
☐☐	
3	

1	
3 5 2	(5+7) tens=
+2 7 1	12 tens=
☐ 2 3	1 hundred+
	2 tens.

3 0 6	3 4 4
+2 4 5	+1 8 2
☐	☐

3 4 7	6 6 1
+2 8 5	+1 8 9
☐	☐

Subtract.

8 5 6	4 7 5
−1 2 4	−2 0 3
☐	☐

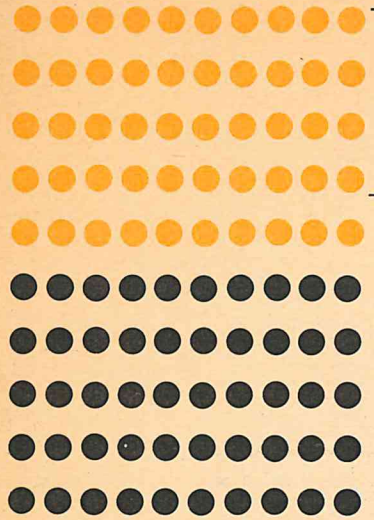
6 12	
8 ~~7~~ ~~2~~	Exchange
−1 2 5	1 ten for
☐	10 ones.

4 11	
~~5~~ ~~1~~ 8	Exchange
−1 4 3	1 hundred
☐	for 10 tens.

7 1 5	6 5 3
−1 2 2	−2 1 8
☐	☐

4 1 0	5 2 2
−1 5 6	−2 4 8
☐	☐

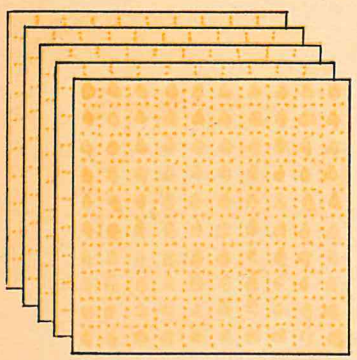
Multiples of Ten or One Hundred



$1 \times 10 = \square$.
$2 \times 10 = \square$.
$3 \times 10 = \square$.
$4 \times 10 = \square$.

$9 \times 10 = \square$.
$7 \times 10 = \square$.
$5 \times 10 = \square$.
$6 \times 10 = \square$.
$8 \times 10 = \square$.

Each sheet of stamps has 100 stamps.



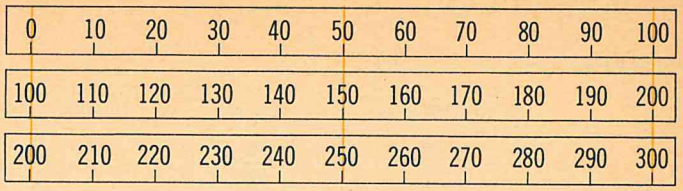
$1 \times 100 = \square$.
$2 \times 100 = \square$.
$3 \times 100 = \square$.
$4 \times 100 = \square$.
$7 \times 100 = \square$.
$6 \times 100 = \square$.
$5 \times 100 = \square$.
$8 \times 100 = \square$.
$9 \times 100 = \square$.

$3 \times (20) = (3 \times 2) \times 10 =$ $6 \times 10 = \square$.
$4 \times (20) = (4 \times 2) \times 10 =$ $\square \times 10 = \square$.
$3 \times 30 = (3 \times 3) \times 10 =$ $\square \times 10 = \square$.
$2 \times 50 = \square$. $2 \times 40 = \square$.
$4 \times 30 = 12$ tens $= 100 + 2$ tens $= \square$.
$6 \times 80 = 48$ tens $= \square + 8$ tens $= \square$.
$8 \times 90 = 72$ tens $= \square + 2$ tens $= \square$.
$8 \times 80 = \square$. $5 \times 40 = \square$.
$6 \times 70 = \square$. $6 \times 60 = \square$.

$2 \times 200 = (2 \times 2) \times 100 = 4 \times 100 = \square$.
$3 \times 200 = (3 \times 2) \times 100 = \square \times 100 = \square$.
$2 \times 300 = \square$ $3 \times 300 = \square$.

10 hundreds = 1 thousand.
20 hundreds = $\square$ thousands.
$5 \times 400 = \square$ hundreds = $\square$ thousands.
$6 \times 400 = 24 \times 100 = \square + 400 = \square$.
$7 \times 800 = 56 \times 100 = \square + 600 = \square$.

The Nearest Hundred



50 is halfway between 0 and 100.
150 is halfway between 100 and $\square$.
250 is halfway between $\square$ and 300.
140 is less than 150. 170 is more than 150.
140 is nearer to 100 than to 200.
170 is nearer to $\square$ than to 100.
220 is less than 250. 275 is more than 250.
220 is nearer to $\square$ than to 300.
275 is nearer to $\square$ than to 200.

Round off to the nearest hundred.

310 is about 300.
460 is about $\square$.
629 is about $\square$.
445 is about $\square$.
382 is about $\square$.
506 is about $\square$.
738 is about $\square$.
852 is about $\square$.

Estimate these sums or differences.

$318 + 590 =$ about $300 + 600 =$ about $\square$.
$440 - 275 =$ about $\square - \square =$ about $\square$.
$140 + 380 =$ about $\square + \square =$ about $\square$.
$280 + 490 =$ about $\square + \square =$ about $\square$.
$613 - 289 =$ about $\square - \square =$ about $\square$.

Thinking Out Multiplication

$4 \times 22 = (4 \times 20) + (4 \times 2) = \square + \square = \square$.
$3 \times 26 = (3 \times 20) + (3 \times 6) = \square + \square = \square$.
$2 \times 43 = (2 \times 40) + (2 \times 3) = \square + \square = \square$.
$3 \times 34 = (3 \times 30) + (3 \times 4) = \square + \square = \square$.
$2 \times 48 = (2 \times 40) + (2 \times 8) = \square + \square = \square$.

$3 \times 123 = (3 \times 100) + (3 \times 20) + (3 \times 3)$
$= \square + \square + \square = \square$.

$4 \times 213 = (4 \times 200) + (4 \times 10) + (4 \times 3)$
$= \square + \square + \square = \square$.

$3 \times 324 = (3 \times 300) + (3 \times 20) + (3 \times 4)$
$= \square + \square + \square = \square$.

A One-Digit Multiplier and a Three-Digit Multiplicand

Multiply 3×231.

Estimate: About $3 \times 200 =$ ☐.

	Hundreds	Tens	Ones
3×231	●●	●●●	●
	●●	●●●	●
	●●	●●●	●

$=$ ☐ hundreds $+$ ☐ tens $+$ ☐ ones.

First Way

$3 \times 1 =$
$3 \times 30 =$
$3 \times 200 =$

$$\begin{array}{r} 231 \\ \times 3 \\ \hline 3 \\ 90 \\ 600 \\ \hline \square\square\square \end{array}$$

Short Way

3×1 one $= 3$ ones

3×3 tens $=$ 9 tens

3×2 hundreds $=$ 6 hundreds.

$$\begin{array}{r} 231 \\ \times 3 \\ \hline 693 \end{array}$$

Multiply 3×125.

Estimate: About $3 \times 100 =$ ☐.

First Way

$3 \times 5 =$
$3 \times 20 =$
$3 \times 100 =$

$$\begin{array}{r} 125 \\ \times 3 \\ \hline 15 \\ 60 \\ 300 \\ \hline \square\square\square \end{array}$$

Short Way

3×5 ones $= 15$ ones

$=$ [1 ten] $+ 5$ ones.

3×2 tens $=$ 6 tens.

6 tens $+$ [1 ten] $=$ 7 tens.

3×1 hundred $=$ 3 hundreds.

$$\begin{array}{r} 125 \\ \times 3 \\ \hline 375 \end{array}$$

Multiply by the first way. Check by addition.

2×311. Estimate: About $2 \times 300 =$ ☐.

$2 \times 1 =$
$2 \times 10 =$
$2 \times 300 =$

$$\begin{array}{r} 311 \\ \times 2 \\ \hline \square \\ \square\square \\ \square\square\square \\ \hline \square\square\square \end{array}$$

Check.

$$\begin{array}{r} 311 \\ +311 \end{array}$$

3×162. Estimate: About $3 \times 200 =$ ☐.

$3 \times 2 =$
$3 \times 60 =$
$3 \times 100 =$

$$\begin{array}{r} 162 \\ \times 3 \\ \hline \square \\ \square\square \\ \square\square\square \\ \hline \square\square\square \end{array}$$

Check.

Multiply by the short way. Then check your answer by multiplying by the first way.

Multiply 6×112.

Estimate: About

$6 \times$ ☐ $=$ ☐.

$$\begin{array}{r} 112 \\ \times 6 \\ \hline \square\square\square \end{array}$$

Check.

$6 \times 2 =$
$6 \times 10 =$
$6 \times 100 =$

$$\begin{array}{r} 112 \\ \times 6 \\ \hline \square \\ \square\square \\ \square\square\square \\ \hline \square\square\square \end{array}$$

Multiply 5×143.

Estimate: About

$5 \times$ ☐ $=$ ☐.

$$\begin{array}{r} 143 \\ \times 5 \\ \hline \square\square\square \end{array}$$

Check.

$5 \times 3 =$
$5 \times 40 =$
$5 \times 100 =$

$$\begin{array}{r} 143 \\ \times 5 \\ \hline \square \\ \square\square \\ \square\square\square \\ \hline \square\square\square \end{array}$$

Multiply 4×109.

Estimate: About

$4 \times$ ☐ $=$ ☐.

$$\begin{array}{r} 109 \\ \times 4 \\ \hline \square\square\square \end{array}$$

Check.

$4 \times 9 =$
$4 \times 00 =$
$4 \times 100 =$

$$\begin{array}{r} 109 \\ \times 4 \\ \hline \square \\ \square\square \\ \square\square\square \\ \hline \square\square\square \end{array}$$

Multiply 7×162.

Estimate: About

$7 \times$ ☐ $=$ ☐.

$$\begin{array}{r} 162 \\ \times 7 \\ \hline \square\square\square\square \end{array}$$

Check.

$7 \times 2 =$
$7 \times 60 =$
$7 \times 100 =$

$$\begin{array}{r} 162 \\ \times 7 \\ \hline \square \\ \square\square \\ \square\square\square \\ \hline \square\square\square\square \end{array}$$

Multiply $3 \times \$3.51$.

Estimate:

About $3 \times \$4.00$

$= \$$ ☐.☐☐.

$$\begin{array}{r} \$3.51 \\ \times 3 \\ \hline \$\square\square.\square\square \end{array}$$

Check.

$3 \times .01 =$
$3 \times .50 =$
$3 \times \$3.00 =$

$$\begin{array}{r} \$3.51 \\ \times 3 \\ \hline .\square\square \\ \square.\square\square \\ \square\square.\square\square \\ \hline \$\square\square.\square\square \end{array}$$

Multiply $4 \times \$2.85$.

Estimate:

About $4 \times \$$ ☐.☐

$= \$$ ☐.☐☐.

$$\begin{array}{r} \$2.85 \\ \times 4 \\ \hline \$\square\square.\square\square \end{array}$$

Check.

$4 \times .05 =$
$4 \times .80 =$
$4 \times \$2.00 =$

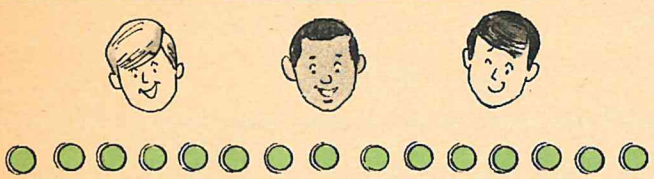
$$\begin{array}{r} \$2.85 \\ \times 4 \\ \hline .\square\square \\ \square.\square\square \\ \square\square.\square\square \\ \hline \$\square\square.\square\square \end{array}$$

Meaning of Division

How many children will 12 cupcakes serve, if each child gets 2 cakes?

The colored lines divide the cakes into groups of 2. There are 6 groups of 2. Six children will be served, because 6 twos = 12, or $6 \times 2 = 12$.

Write: $12 \div 2 = 6$, or $2\overline{)12}^{\,6}$



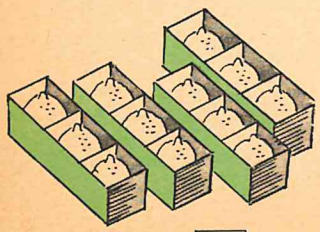
These 3 boys ran an errand together, and were paid 15¢. To share the 15¢ equally, each boy takes one penny, then each boy takes another penny, and so on, until all the pennies are taken. Show each choice by drawing a line from each boy to a penny. How many pennies does each boy get? Each boy gets 5¢, because $3 \times 5 = 15$.

Write: $15 \div 3 = 5$, or $3\overline{)15}^{\,5}$

Complete each multiplication example shown below. Then write two division examples that use the same numbers.

$2 \times 4 = 8.$	$8 \div 2 = 4.$	$8 \div 4 = 2.$
$3 \times 7 = 21.$	$21 \div \square = \square.$	$21 \div \square = \square.$
$5 \times 9 = \square.$	$\square \div 5 = \square.$	$\square \div 9 = \square.$
$7 \times 8 = \square.$	$\square \div 7 = \square.$	$\square \div 8 = \square.$
$3 \times 10 = \square.$	$\square \div \square = \square.$	$\square \div \square = \square.$
$4 \times 20 = \square.$	$\square \div \square = \square.$	$\square \div \square = \square.$

How many boxes will 21 lemons fill, if each box holds 3 lemons?

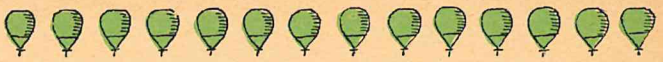


Answer: ☐ boxes

A class of 20 children was divided into 5 relay teams. How many children were on each team?

Answer: ☐ children

Division with a Remainder



Draw lines to divide these 14 tops into groups of 3.

There are ☐ groups of 3, and ☐ tops are left.

☐ and ☐ left.

Write: $3\overline{)14}^{\,\square}$

☐ and ☐ left — $7\overline{)18}^{\,\square}$

☐ and ☐ left — $8\overline{)65}^{\,\square}$

☐ and ☐ left — $5\overline{)21}^{\,\square}$

☐ and ☐ left — $7\overline{)51}^{\,\square}$

☐ and ☐ left — $6\overline{)20}^{\,\square}$

☐ and ☐ left — $10\overline{)72}^{\,\square}$

☐ and ☐ left — $9\overline{)75}^{\,\square}$

☐ and ☐ left — $6\overline{)51}^{\,\square}$

How many 5¢ pencils can you buy for 23¢?

☐ pencils and ☐ ¢ left

How many weeks are there in 30 days?

☐ weeks and ☐ days left

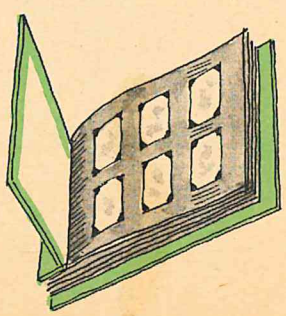
Five boys shared 17¢. How much did each boy get?

☐ ¢ each and ☐ ¢ left

Eight girls shared 43 jacks. How many did each girl get?

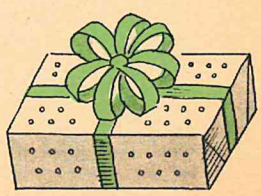
☐ each and ☐ left

John is putting 75 photographs into an album. Each page has room for 6 photographs. How many pages were filled?



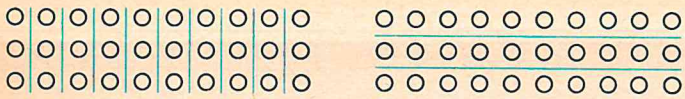
☐ pages filled and ☐ photographs left

Mary is making rosettes to put on the gifts she is wrapping. Each rosette uses 20 inches of ribbon. If she has 84 inches of ribbon, how many rosettes can she make?



☐ rosettes and ☐ inches of ribbon left

Multiplying by Ten



10 columns of threes = 3 rows of tens.

$10 \times 3 = 3 \times 10 = 3$ tens $= 30$.

$10 \times 4 = 4 \times 10 = 4$ tens $= 40$.

$10 \times 5 = \square \times 10 = \square$.

$10 \times 2 = \square \times 10 = \square$.

$10 \times 1 = \square \times 10 = \square$.

$10 \times 8 = \square \times 10 = \square$.

$10 \times 6 = \square \times 10 = \square$.

$10 \times 7 = \square \times 10 = \square$.

	Tens	Ones
10 ×		4
=	4	0

Multiplying 4 by 10 changes 4 ones to 4 tens. The 4 moves one place to the left, and 0 holds the ones place.

	Tens	Ones
10 ×		9
=	9	0

	Tens	Ones
10 ×		6
=		

	Tens	Ones
10 ×		5
=		

$10 \times 23 = 23 \times 10 = 23$ tens

$= 20$ tens $+ 3$ tens $= 2$ hundreds $+ 3$ tens $= 230$.

	Hundreds	Tens	Ones
10 ×		2	3
=	2	3	0

Multiplying a number by 10 moves each digit one place to the left, and 0 holds the ones place.

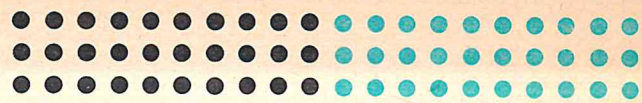
10 ×		4	6
=			

10 ×		8	3
=			

10 ×		7	5
=			

10 ×		6	9
=			

Multiplying by a Multiple of Ten



$20 \times 3 = 2 \times (10 \times 3) = 2 \times 30 = 60$.

Multiplying by 20 is like multiplying first by 10, and then by 2.

$\begin{array}{r} 3 \\ \times 20 \\ \hline 60 \end{array}$

To multiply by 20,

first put down 0, and move left.

This is like multiplying by 10.

Then multiply by 2.

$30 \times 6 = 3 \times (10 \times 6) = 3 \times \square = \square$.

Multiplying by 30 is like multiplying first by 10 and then by 3.

$\begin{array}{r} 6 \\ \times 30 \\ \hline \square\square 0 \end{array}$

Put down 0.

Move left, and multiply by 3.

$\begin{array}{r} 8 \\ \times 40 \\ \hline \square\square \end{array}$

Put down 0.

Move left, and multiply by 4.

$\begin{array}{r} 7 \\ \times 60 \\ \hline \square\square \end{array}$

Put down 0.

Move left, and multiply by 6.

$\begin{array}{r} 31 \\ \times 20 \\ \hline \square\square \end{array}$

Put down 0.

Move left, and multiply by 2.

$\begin{array}{r} 45 \\ \times 40 \\ \hline \square\square \end{array}$

Put down 0.

Move left, and multiply by 4.

$\begin{array}{r} 15 \\ \times 60 \\ \hline \square\square \end{array}$

$\begin{array}{r} 24 \\ \times 50 \\ \hline \square\square \end{array}$

$\begin{array}{r} 36 \\ \times 30 \\ \hline \square\square \end{array}$

$\begin{array}{r} 81 \\ \times 20 \\ \hline \square\square \end{array}$

$\begin{array}{r} 64 \\ \times 30 \\ \hline \square\square \end{array}$

$\begin{array}{r} 72 \\ \times 10 \\ \hline \square\square \end{array}$

$\begin{array}{r} 37 \\ \times 50 \\ \hline \square\square \end{array}$

$\begin{array}{r} 28 \\ \times 60 \\ \hline \square\square \end{array}$

$\begin{array}{r} 42 \\ \times 40 \\ \hline \square\square \end{array}$

$\begin{array}{r} 35 \\ \times 30 \\ \hline \square\square \end{array}$

$\begin{array}{r} 63 \\ \times 70 \\ \hline \square\square \end{array}$

$\begin{array}{r} 16 \\ \times 50 \\ \hline \square\square \end{array}$

$20 \times 4 = \square\square$. $30 \times 7 = \square\square$. $40 \times 8 = \square\square$.

A Two-Digit Multiplier and a Two-Digit Multiplicand

Multiply 32 × 24. The multiplier 32 = 30 + 2.

Multiply by 2.	Multiply by 30.	Then add.
24 × 2 = 48	24 × 30 = 720	48 + 720 = ☐
Step I	Step II	Step III

To check the answer, interchange the multiplier and multiplicand, and then multiply in the same way.

Multiply 24 × 32. The multiplier 24 = 20 + 4.

32 × 4 = ☐	32 × 20 = ☐	+ ☐
Step I	Step II	Step III

Short Way of Arranging Your Work:
Step I—Multiply by 1.
Step II—Multiply by 40.
Step III—Add.

26 × 41 = 26, 1040, ☐

Check. 41 × 26 = ☐, ☐, ☐

Multiply and check. Arrange your work in the short form used above.

13 × 12	16 × 16
21 × 23	32 × 35
41 × 14	57 × 21

Multiply. Then check by interchanging the multiplier and multiplicand.

28 × 17: The multiplier 28 = 20 + 8.

17 × 8 = ☐	17 × 20 = ☐	+ ☐

17 × 28: The multiplier 17 = 10 + 7.

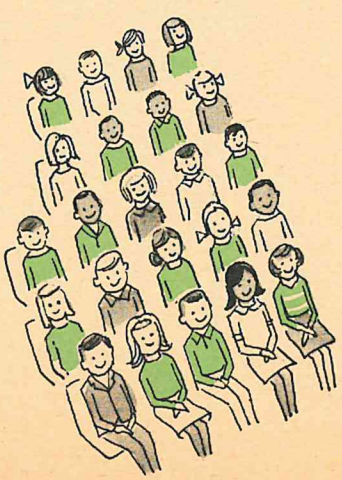
28 × 7 = ☐	28 × 10 = ☐	+ ☐

48 × 35: The multiplier 48 = 40 + 8.

35 × 8 = ☐	35 × 40 = ☐	+ ☐

35 × 48: The multiplier 35 = 30 + 5.

48 × 5 = ☐	48 × 30 = ☐	+ ☐



An auditorium has 24 rows, with 18 seats in each row. How many seats are there in the auditorium?

Answer: ☐

A One-Digit Divisor and a Two-Digit Dividend

How many threes are there in 72?

Step I—Trying tens
$10 \times 3 = 30$.
$20 \times 3 = 60$.
$30 \times 3 = 90$ (too much).

Step II—Estimate:
more than 20 threes

Step III

$$\begin{array}{r} 20 \\ 3\overline{)72} \\ 20 \times 3 = 60 \\ 72 - 60 = 12 \end{array}$$

Step IV

$$\begin{array}{r} 4 \\ 20 \\ 3\overline{)72} \\ 60 \\ 12 \\ 4 \times 3 = 12 \\ 12 - 12 = 0 \end{array}$$

Step V $20 + 4 = 24$.

There are 24 threes in 72.

Check. $24 \times 3 = 3 \times 24$.

24×3

Compare with the dividend.

How many fours are there in 70?

Step I—Trying tens
$10 \times 4 = 40$.
$20 \times 4 = 80$ (too much).

Step II—Estimate:
more than 10

Step III

$$\begin{array}{r} 10 \\ 4\overline{)70} \\ 10 \times 4 = 40 \\ 70 - 40 = 30 \end{array}$$

Step IV

$$\begin{array}{r} 7 \\ 10 \\ 4\overline{)70} \\ 40 \\ 30 \\ 7 \times 4 = 28 \\ 2 \end{array}$$

Answer:
17 fours and 2 left

Check. $17 \times 4 = 4 \times 17 = 68$.

$68 + 2$

Compare with the dividend.

How many twos are there in 96?

$10 \times 2 =$ ☐. $20 \times 2 =$ ☐.
$30 \times 2 =$ ☐. $40 \times 2 =$ ☐.
$50 \times 2 =$ ☐ (too much).

☐☐ = estimate
2)96
= estimate × 2
96 − ☐ = ☐☐
= ☐ twos
☐☐ + ☐ = ☐ Answer

How many fives are there in 80?

$10 \times 5 =$ ☐. $20 \times 5 =$ ☐.

☐☐ = estimate
5)80
= estimate × 5
80 − ☐ = ☐☐
= ☐ fives
☐☐ + ☐ = ☐ Answer

How many sixes are there in 75?

$10 \times 6 =$ ☐. $20 \times 6 =$ ☐.

☐☐ = estimate
6)75
75 − ☐ = ☐☐
= ☐ sixes
☐☐ sixes and ☐ left

$10 \times 2 =$ ☐. $20 \times 2 =$ ☐.
$30 \times 2 =$ ☐. $40 \times 2 =$ ☐.
$50 \times 2 =$ ☐.

2)85
Answer:
☐
and ☐ left

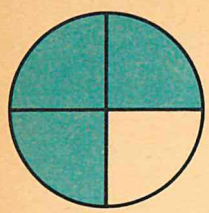
$10 \times 3 =$ ☐. $20 \times 3 =$ ☐.
$30 \times 3 =$ ☐.

3)69
Answer:
☐

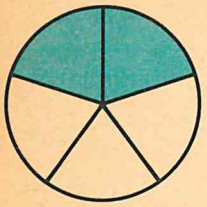
$10 \times 7 =$ ☐. $20 \times 7 =$ ☐.

7)93
Answer:
☐
and ☐ left

Meaning of Fractions

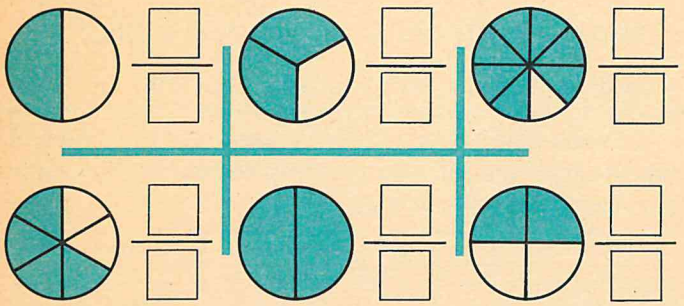


Divide a whole into 4 equal parts → $\frac{3}{4}$ ← and take 3 parts.

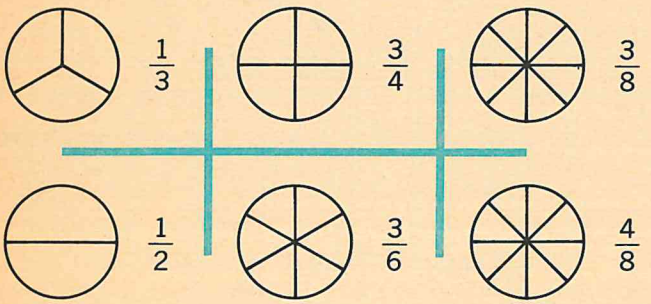


Divide a whole into ☐ equal parts → $\frac{2}{5}$ ← and take ☐ parts.

Write the fraction that shows what part of a whole is colored in each diagram.



Color part of a whole to show the meaning of each of these fractions.



1 whole							
$\frac{1}{2}$	$\frac{1}{2}$						
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$					
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$				
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$		
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

In each line at the right, circle the two fractions that are equal to each other.

$\frac{1}{2}$	$\frac{2}{4}$	$\frac{2}{6}$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{2}{6}$
$\frac{2}{3}$	$\frac{4}{6}$	$\frac{5}{6}$
$\frac{3}{4}$	$\frac{6}{8}$	$\frac{7}{8}$
$\frac{3}{6}$	$\frac{1}{2}$	$\frac{1}{4}$
$\frac{4}{8}$	$\frac{3}{6}$	$\frac{4}{6}$

Circle the right word in each sentence.

$\frac{1}{2}$ is (larger smaller) than $\frac{1}{3}$.

$\frac{1}{4}$ is (larger smaller) than $\frac{1}{3}$.

$\frac{1}{4}$ is (larger smaller) than $\frac{1}{5}$.

The more equal parts there are in a whole, the (smaller larger) each part is.

If the numerator of a fraction is 1, the larger the denominator is, the (smaller larger) the fraction is.

Rewrite the fractions in each line in order of size, with the smallest fraction first.

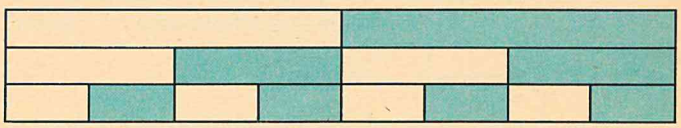
$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$ _____

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ _____

$\frac{1}{2}, \frac{2}{6}, \frac{2}{3}$ _____

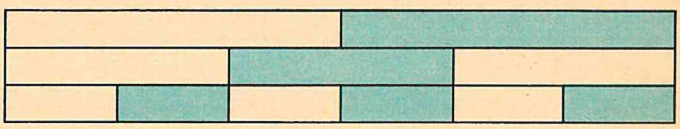
$\frac{4}{8}, \frac{3}{4}, \frac{2}{3}$ _____

Compare halves, fourths, and eighths.



$\frac{1}{2} = \frac{\square}{4} = \frac{\square}{8}$. $\frac{1}{4} = \frac{\square}{8}$. $\frac{3}{4} = \frac{\square}{8}$.

Compare halves, thirds, and sixths.



$\frac{1}{2} = \frac{\square}{6}$. $\frac{1}{3} = \frac{\square}{6}$. $\frac{2}{3} = \frac{\square}{6}$.

In some of the sentences below, the word half or twice is missing. Fill in the missing word.

A fourth is half as big as a half.

A fourth is twice as big as an eighth.

A third is _____ as big as a sixth.

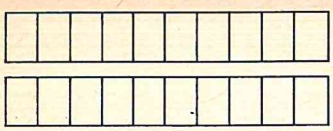
An eighth is _____ as big as a fourth.

A sixth is _____ as big as a third.

A half is _____ as big as a fourth.

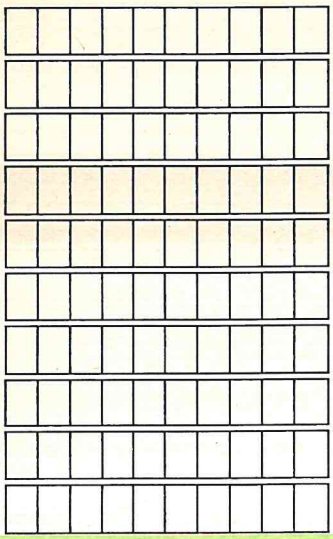
Tenths

If a bar is divided into 10 equal parts, each part is called one tenth. $\frac{1}{10}$ means one tenth.



There are ☐ tenths in 1 whole. $\frac{\square}{10} = 1$.

There is a fraction next to each box below. Color enough tenths to show what the fraction means.

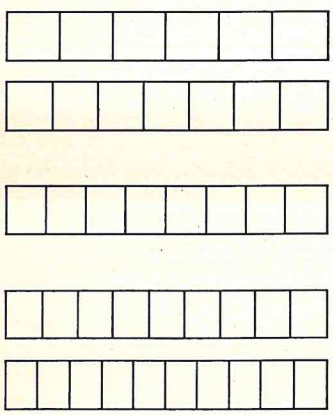


$\frac{1}{10}$ = 1 tenth.
$\frac{2}{10}$ = 2 tenths.
$\frac{3}{10}$ = ☐ tenths.
$\frac{4}{10}$ = ☐ tenths.
$\frac{5}{10}$ = ☐ tenths.
$\frac{6}{10}$ = ☐ tenths.
$\frac{7}{10}$ = ☐ tenths.
$\frac{8}{10}$ = ☐ tenths.
$\frac{9}{10}$ = ☐ tenths.
$\frac{10}{10}$ = ☐ tenths = 1 whole

To get $\frac{3}{10}$ of a whole, divide the whole into ☐ equal parts and then take ☐ of them.

To get $\frac{5}{10}$ of a whole, divide the whole into ☐ equal parts and then take ☐ of them.

To get $\frac{7}{10}$ of a whole, divide the whole into ☐ equal parts and then take ☐ of them.

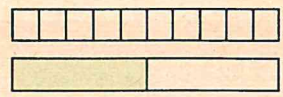


This whole is divided into sixths. Color 1 sixth.
This whole is divided into sevenths. Color 1 seventh.
This whole is divided into eighths. Color 1 eighth.
This whole is divided into ninths. Color 1 ninth.
This whole is divided into tenths. Color 1 tenth.

Circle the right word in each sentence.

$\frac{1}{6}$ is (larger smaller) than $\frac{1}{7}$.
$\frac{1}{8}$ is (larger smaller) than $\frac{1}{10}$.
$\frac{1}{9}$ is (larger smaller) than $\frac{1}{8}$.
$\frac{1}{10}$ is (larger smaller) than $\frac{1}{9}$.
$\frac{1}{7}$ is (larger smaller) than $\frac{1}{10}$.

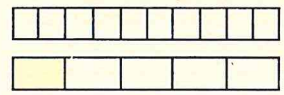
Tenths, Halves, and Fifths



This bar is divided into tenths.
This bar is divided into halves.

In the first bar, color enough tenths to equal 1 half.

☐ tenths = 1 half. $\frac{\square}{10} = \frac{1}{2}$.



This bar is divided into tenths.
This bar is divided into fifths.

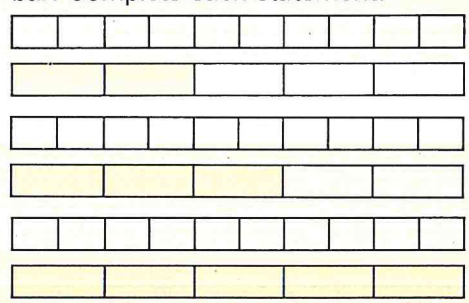
In the first bar, color enough tenths to equal 1 fifth

☐ tenths = 1 fifth. $\frac{\square}{10} = \frac{1}{5}$.

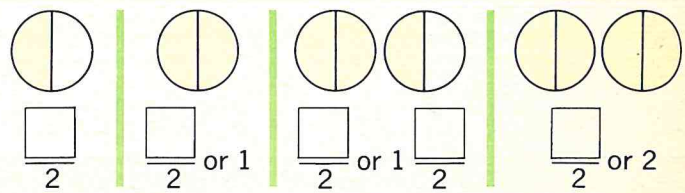
Circle the right word to complete the sentence:

1 tenth is (half twice) as large as 1 fifth.

In each pair of bars, color enough tenths in the first bar to equal the number of fifths shown in the second bar. Complete each statement.



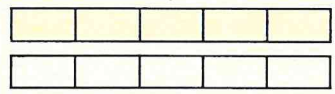
$\frac{\square}{10} = \frac{2}{5}$.
$\frac{\square}{10} = \frac{3}{5}$.
$\frac{\square}{10} = \frac{5}{5} = 1$.



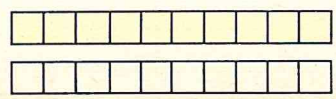
$\frac{\square}{2}$ | $\frac{\square}{2}$ or 1 | $\frac{\square}{2}$ or 1 $\frac{\square}{2}$ | $\frac{\square}{2}$ or 2

Count by halves.

$\frac{1}{2}$, 1, $1\frac{1}{2}$, ☐, ☐, $\frac{\square}{\square}$, ☐, ☐, $\frac{\square}{\square}$.



Color 1 more fifth at a time, and complete these statements: $\frac{6}{5} = 1\frac{\square}{5}$, $\frac{7}{5} = 1\frac{\square}{5}$, $\frac{8}{5} = 1\frac{\square}{5}$, $\frac{10}{5} = \square$.



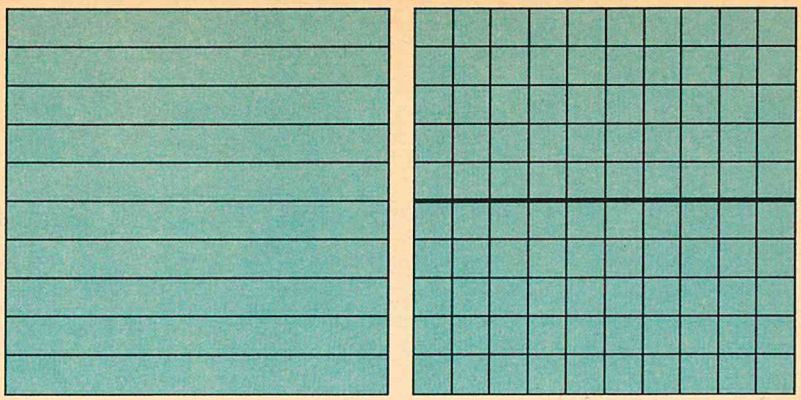
Color 1 more tenth at a time, and complete these statements: $\frac{11}{10} = 1\frac{\square}{10}$, $\frac{12}{10} = 1\frac{\square}{10}$, $\frac{13}{10} = 1\frac{\square}{\square}$.

Hundredths

The first square is divided into ten equal parts. Each part is one tenth of the square.

In the second square, each tenth is divided into ten equal parts. Each row of small squares is one tenth of the large square. There are ten rows of small squares. There are ten small squares in each row.

$10 \times 10 = 100$.



There are 100 small squares in the second square. Each small square is one hundredth of that square.

$\frac{1}{100}$ = one hundredth.

How many small squares are there in 1 row?

Answer: $\square$. $\frac{\square}{100} = \frac{1}{10}$.

How many small squares are there in 2 rows?

Answer: $\square$. $\frac{\square}{100} = \frac{2}{10}$.

How many small squares are there in 3 rows?

Answer: $\square$. $\frac{\square}{100} = \frac{3}{10}$.

How many small squares are there in 4 rows?

Answer: $\square$. $\frac{\square}{100} = \frac{4}{10}$.

Count the small squares by tens, 1 row at a time. Then complete these statements:

$\frac{\square}{100} = \frac{5}{10}$. $\frac{\square}{100} = \frac{6}{10}$. $\frac{\square}{100} = \frac{7}{10}$.

$\frac{\square}{100} = \frac{8}{10}$. $\frac{\square}{100} = \frac{9}{10}$. $\frac{\square}{100} = \frac{10}{10} = 1$.

Write the number of tenths that each fraction is equal to.

$\frac{30}{100} = \frac{\square}{10}$. $\frac{90}{100} = \frac{\square}{10}$. $\frac{50}{100} = \frac{\square}{10}$.

$\frac{20}{100} = \frac{\square}{10}$. $\frac{40}{100} = \frac{\square}{10}$. $\frac{10}{100} = \frac{\square}{10}$.

$\frac{80}{100} = \frac{\square}{10}$. $\frac{60}{100} = \frac{\square}{10}$. $\frac{100}{100} = \frac{\square}{10}$.

The dark black line divides the second large square in half. There are $\square$ rows above the line. $\frac{\square}{10} = \frac{1}{2}$.

There are $\square$ squares above the line. $\frac{\square}{100} = \frac{1}{2}$.

A dollar is worth as much as ten dimes.

A dime is worth $\frac{1}{10}$ of a dollar.

2 dimes are worth $\frac{\square}{10}$ of a dollar.

3 dimes are worth $\frac{\square}{10}$ of a dollar.

5 dimes are worth $\frac{\square}{10}$ or $\frac{\square}{2}$ of a dollar.

9 dimes are worth $\frac{\square}{10}$ of a dollar.

A dime is worth ten cents.

Ten dimes are worth $\square \times 10$ cents or 100 cents.

A dollar is worth $\square$ cents.

1 cent is $\frac{1}{100}$ of a dollar. 10 cents is $\frac{10}{100}$ of a dollar.

20 cents is $\frac{\square}{100}$ of a dollar.

30 cents is $\frac{\square}{100}$ of a dollar.

80 cents is $\frac{\square}{100}$ of a dollar.

50 cents is $\frac{\square}{100}$ of a dollar.

60 cents is $\frac{\square}{100}$ of a dollar.

90 cents is $\frac{\square}{100}$ of a dollar.

$\frac{10}{100}$ of a dollar = $\square$ cents.

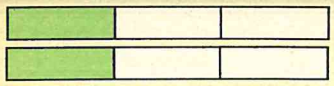
$\frac{30}{100}$ of a dollar = $\square$ dimes = $\frac{\square}{10}$ of a dollar.

$\frac{60}{100}$ of a dollar = $\square$ dimes = $\frac{\square}{10}$ of a dollar.

$\frac{90}{100}$ of a dollar = $\square$ dimes = $\frac{\square}{10}$ of a dollar.

Fractions and Division

If two candy bars are divided into 3 equal shares, how big is each share? $\frac{1}{3}$ of 2 = ?

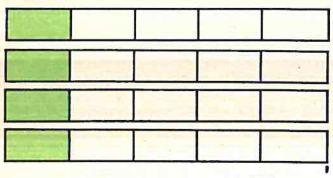


Let each rectangle stand for 1 bar.

First divide each bar into 3 equal parts. Then take $\frac{1}{3}$ of each bar. $\frac{1}{3}$ and $\frac{1}{3} = \frac{2}{3}$.

Then $\frac{1}{3}$ of 2 $= \frac{2}{3}$. $2 \div 3 = \frac{2}{3}$.

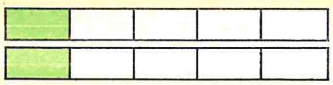
If four bars are divided into 5 equal shares, how big is each share? $\frac{1}{5}$ of 4 = ?



Divide each bar into 5 equal parts. $\frac{1}{5}$ and $\frac{1}{5}$ and $\frac{1}{5}$ and $\frac{1}{5} = \frac{4}{5}$.

Then $\frac{1}{5}$ of 4 $= \frac{\square}{5}$. $4 \div 5 = \frac{4}{5}$.

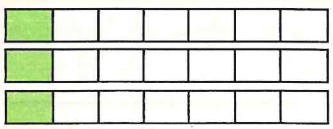
Find $2 \div 5$, or $\frac{1}{5}$ of 2.



Divide each bar into 5 equal parts.

$\frac{1}{5}$ and $\frac{1}{5} = \frac{\square}{5}$. $\frac{1}{5}$ of $2 = \frac{\square}{5}$. $2 \div 5 = \frac{\square}{5}$.

Find $3 \div 7$, or $\frac{1}{7}$ of 3.

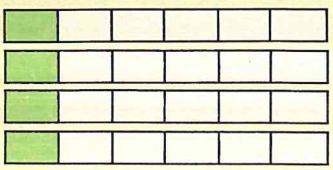


Draw $\square$ bars.

Divide each bar into $\square$ equal parts.

$\frac{1}{7}$ and $\frac{1}{7}$ and $\frac{1}{7} = \frac{\square}{7}$. $\frac{1}{7}$ of $3 = \square$. $3 \div 7 = \frac{\square}{7}$.

Find $4 \div 6$, or $\frac{1}{6}$ of 4.

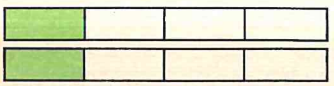


Draw $\square$ bars.

Divide each bar into $\square$ equal bars.

$\frac{1}{6}$ of $4 = \frac{\square}{6}$. $4 \div 6 = \frac{\square}{6}$.

Find $2 \div 4$, or $\frac{1}{4}$ of 2.



Draw $\square$ bars.

Divide each bar into $\square$ equal parts.

$\frac{1}{4}$ and $\frac{1}{4} = \frac{\square}{4}$. $\frac{1}{4}$ of $2 = \frac{\square}{4}$.

In $2 \div 3$, 2 is the dividend, and 3 is the divisor.
In $\frac{2}{3}$, 2 is the numerator, and 3 is the denominator.

In $4 \div 5$, $\square$ is the dividend, and $\square$ is the divisor.
In $\frac{4}{5}$, $\square$ is the numerator, and $\square$ is the denominator.

In $2 \div 5$, $\square$ is the dividend, and $\square$ is the divisor.
In $\frac{2}{5}$, $\square$ is the numerator, and $\square$ is the denominator.

$2 \div 3 = \frac{2}{3}$. The dividend 2 is the numerator of the fraction. The divisor 3 is the denominator of the fraction.

$4 \div 5 = \frac{4}{5}$. The dividend 4 is the numerator of the fraction. The divisor 5 is the denominator of the fraction.

$2 \div 5 = \frac{2}{5}$. The dividend 2 is the numerator of the fraction. The divisor 5 is the denominator of the fraction.

To divide one number by another, we may write the answer as a fraction. The dividend is the numerator of the fraction. The divisor is the denominator.

$3 \div 5 = \frac{\square}{5}$. $4 \div 9 = \frac{\square}{9}$.

$7 \div 8 = \frac{7}{\square}$. $5 \div 6 = \frac{5}{\square}$.

$3 \div 8 = \frac{\square}{\square}$. $4 \div 7 = \frac{\square}{\square}$.

$5 \div 10 = \frac{\square}{\square}$. $4 \div 2 = \frac{\square}{\square}$.

$2 \div 3 = \frac{2}{3}$. If this statement is read backwards, it says, $\frac{2}{3} = 2 \div 3$.

$4 \div 5 = \frac{4}{5}$. So $\frac{4}{5} = 4 \div 5$.

$2 \div 5 = \frac{2}{5}$. So $\frac{2}{5} = 2 \div 5$.

A fraction may be thought of as a division example. The fraction line is like the division sign. The numerator is the dividend. The denominator is the divisor.

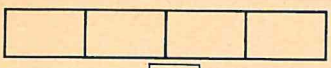
$\frac{3}{8} = \square \div \square$. $\frac{5}{9} = \square \div \square$.

$\frac{4}{10} = \square \div \square$. $\frac{5}{6} = \square \div \square$.

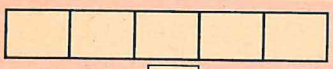
Comparing Whole and Part

This bar is divided into 2 equal parts. Each part is $\frac{1}{2}$ as long as the whole. The whole is 2 times as long as each part.

This bar is divided into 3 equal parts. Each part is $\frac{1}{3}$ as long as the whole. The whole is 3 times as long as each part.

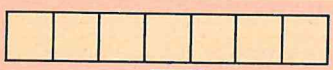


This bar is divided into ☐ equal parts. Each part is ☐ as long as the whole. The whole is ☐ times as long as each part.



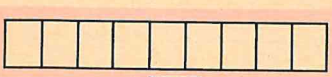
This bar is divided into ☐ equal parts. Each part is ☐ as long as the whole. The whole is ☐ times as long as each part.

This bar is divided into ☐ equal parts. Each part is ☐ as long as the whole. The whole is ☐ times as long as each part.

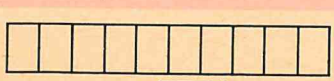


This bar is divided into ☐ equal parts. Each part is ☐ as long as the whole. The whole is ☐ times as long as each part.

This bar is divided into ☐ equal parts. Each part is ☐ as long as the whole. The whole is ☐ times as long as each part.



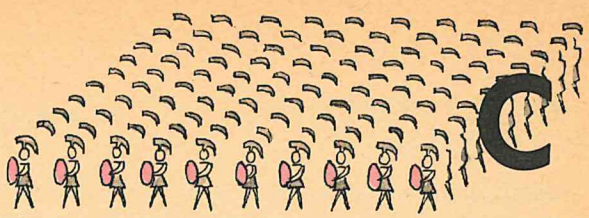
This bar is divided into ☐ equal parts. Each part is ☐ as long as the whole. The whole is ☐ times as long as each part.



This bar is divided into ☐ equal parts. Each part is ☐ as long as the whole. The whole is ☐ times as long as each part.

This bar is divided into ☐ equal parts. Each part is ☐ as long as the whole. The whole is ☐ times as long as each part.

Roman Numerals



In Roman numerals, C means 100.

I means ☐. V means ☐.

X means ☐. L means ☐.

When a Roman numeral is repeated, you add.

II = 1 + 1 = ☐. III = 1 + 1 + 1 = ☐.

XX = 10 + 10 = ☐. XXX = 10 + 10 + 10 = ☐.

CC = 100 + 100 = ☐.

CCC = 100 + 100 + 100 = ☐.

In Roman numerals, when a small number follows a larger number, you add them.

XI = ☐ + 1 = ☐. LX = ☐ + 10 = ☐.

CX = ☐ + 10 = ☐.

CL = 100 + ☐ = ☐.

In Roman numerals, when a small number is written before a larger one, you subtract them.

IV = 5 − 1 = ☐. IX = ☐ − 1 = ☐.

XL = ☐ − 10 = ☐. XC = 100 − ☐ = ☐.

Write in Arabic numerals.

CCI = ☐. CCV = ☐.

CCX = ☐. CCCI = ☐.

CCCV = ☐. CCCX = ☐.

CLX = ☐. CXL = ☐.

CLXX = ☐. CLXXX = ☐.

CLXXXV = ☐. CCLXI = ☐.

Write in Roman numerals.

125 = ☐. 215 = ☐.

350 = ☐. 107 = ☐.

170 = ☐. 112 = ☐.

251 = ☐. 95 = ☐.

190 = ☐. 195 = ☐.

290 = ☐. 295 = ☐.

The Commutative Law of Addition

Here are 5 colored beads and 2 black beads. 5 + 2

Because the colored beads are on the left, and the black beads are on the right, you write 5 + 2, with the 5 on the left and the 2 on the right.

If you move the colored beads to the other side of the black beads, you have

2 black beads and 5 colored beads. 2 + 5

Then the 2 is on the left and the 5 is on the right. The beads have changed places, but the number of beads is the same.

So, $5 + 2 = 2 + 5$.

When you write the sum of two numbers, and then let the two addends change places, the sum remains the same. This is known as the **commutative law of addition.**

Here are some more examples of the commutative law of addition. To show the meaning of a sum written near a line of beads, use your crayon to show some beads in color and some beads in black as in the example above. Fill in the missing numbers.

6 + 9 ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

9 + 6 ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

6 + 9 = 9 + 6.

8 + 3 ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

3 + 8 ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

8 + 3 = 3 + 8.

5 + 7 ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

7 + 5 ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

5 + 7 = 7 + ☐.

$100 + 100 = 100 +$ ☐. $76 + 12 =$ ☐ + ☐.

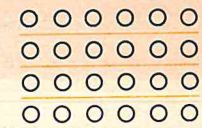
$50 + 20 =$ ☐ + 50. $43 + 9 =$ ☐ + ☐.

$100 + 1 =$ ☐ + ☐. $2 + 70 =$ ☐ + ☐.

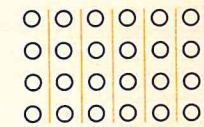
$95 + 6 =$ ☐ + 95. $800 + 5 =$ ☐ + ☐.

The Commutative Law of Multiplication

Here are some beads arranged in equal rows. There are 6 beads in each row. There are 4 rows of beads. The number of beads is 4×6.



The same beads are also arranged in columns. There are 4 beads in each column. There are 6 columns. So the number of beads is 6×4.



Then $4 \times 6 = 6 \times 4$.

When you write the product of two numbers, and then let the multiplicand and multiplier change places, the product remains the same. This is known as the **commutative law of multiplication.**

We used the commutative law of multiplication on page 6 when we said $10 \times 3 = 3 \times 10$. We used it again on page 7 when we said 17×28 and 28×17 gave the same product.

Below are some more examples of the commutative law of multiplication. To show the meaning of each product written near a set of beads, draw lines that divide the beads either into rows or columns, as in the example above. Fill in the missing numbers.

3 × 4 ○ ○ ○ ○ / ○ ○ ○ ○ / ○ ○ ○ ○ 4 × 3 ○ ○ ○ ○ / ○ ○ ○ ○ / ○ ○ ○ ○

$3 \times 4 = 4 \times 3$.

5 × 2 ○ ○ ○ ○ ○ / ○ ○ ○ ○ ○ 2 × 5 ○ ○ ○ ○ ○ / ○ ○ ○ ○ ○

$5 \times 2 = 2 \times$ ☐.

3 × 6 ○ ○ ○ ○ ○ ○ / ○ ○ ○ ○ ○ ○ / ○ ○ ○ ○ ○ ○ 6 × 3 ○ ○ ○ ○ ○ ○ / ○ ○ ○ ○ ○ ○ / ○ ○ ○ ○ ○ ○

$3 \times 6 =$ ☐ × ☐.

Four-Digit Numerals

Thousands	Hundreds	Tens	Ones	
2	4	2	8	= 2 thousands + 4 hundreds + 2 tens + 8 ones.
5	3	0	7	= ☐ thousands + ☐ hundreds + ☐ tens + ☐ ones.
2	6	4	0	= ☐ thousands + ☐ hundreds + ☐ tens + ☐ ones.
3	0	1	2	= ☐ thousands + ☐ hundreds + ☐ ten + ☐ ones.

In a four-digit numeral, the fourth digit from the right shows the number of thousands.

Write each of these numbers in numerals.

one thousand three hundred twenty-five				
five thousand eight hundred				
three thousand sixty				
two thousand four hundred one				
seven thousand eight				

Write in the rings as you count

by ones: 2,998 ☐ ☐ ☐

by hundreds: 4,800 ☐ ☐ ☐

by thousands: 6,000 ☐ ☐ ☐

by five hundreds: 3,500 ☐ ☐ ☐

1 thousand = 10 hundreds. 50 hundreds = 5 thousands.

2 thousands = ☐ hundreds. 60 hundreds = ☐ thousands.

3 thousands = ☐ hundreds. 70 hundreds = ☐ thousands.

4 thousands = ☐ hundreds. 80 hundreds = ☐ thousands.

Exchange 10 hundreds for 1 thousand.

4 thousands + 12 hundreds = 5 thousands + 2 hundreds.

2 thousands + 16 hundreds = 3 thousands + ☐ hundreds.

1 thousand + 14 hundreds = ☐ thousands + 4 hundreds.

3 thousands + 15 hundreds = ☐ thousands + ☐ hundreds.

6 thousands + 10 hundreds = ☐ thousands + ☐ hundreds.

Exchange 1 thousand for 10 hundreds.

2 thousands + 7 hundreds = 1 thousand + 17 hundreds.

6 thousands + 3 hundreds = 5 thousands + ☐ hundreds.

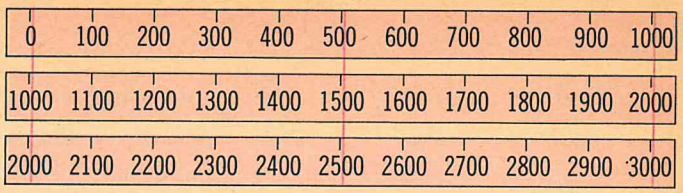
8 thousands + 1 hundred = ☐ thousands + 11 hundreds.

4 thousands + 2 hundreds = ☐ thousands + ☐ hundreds.

7 thousands + 0 hundreds = ☐ thousands + ☐ hundreds.

9 thousands + 4 hundreds = ☐ thousands + ☐ hundreds.

The Nearest Thousand



500 is halfway between 0 and 1000.

1500 is halfway between 1000 and ☐.

2500 is halfway between ☐ and 3000.

1300 is less than 1500. 1700 is more than 1500.

1300 is nearer to ☐ than to 2000.

1700 is nearer to ☐ than to 1000.

2400 is less than 2500. 2850 is more than 2500.

2400 is nearer to ☐ than to 3000.

2850 is nearer to ☐ than to 2000.

Round off to the nearest thousand.

4800 is about 5000.

2350 is about ☐.

1789 is about ☐.

6199 is about ☐.

7400 is about ☐.

8800 is about ☐.

Estimate these sums or differences.

5600 + 2100 = about 6000 + 2000 = about ☐.

4900 − 2400 = about ☐ − ☐ = about ☐.

2120 + 3189 = about ☐ + ☐ = about ☐.

8210 − 4599 = about ☐ − ☐ = about ☐.

Add.

4214	5123
+3102	+2465
☐	☐

Subtract.

8764	5864
−1231	−3252
☐	☐

1
5812
+1435
☐2☐☐

12 hundreds = 1 thousand + 2 hundreds

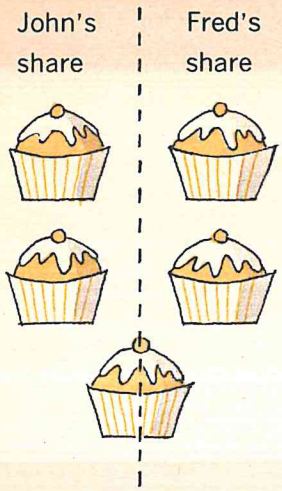
5 12
6 2 8 7
−1532
☐

Exchange 1 thousand for 10 hundreds.

2710	3681
+4823	+1653
☐	☐

7076	9162
−2431	−1825
☐	☐

Remainder Expressed as a Fraction



John and Fred have 5 cupcakes to share equally. First each of them took 2 cupcakes. One cupcake was left. They had to divide this cake into 2 equal parts. $1 \div 2 = \frac{1}{2}$. Each of them took half of the cupcake. Then each one had $2\frac{1}{2}$ cupcakes.

To divide 5 by 2, you used to write:

$$\begin{array}{r} 2 \\ 2\overline{)5} \\ \underline{4} \\ 1 \end{array}$$ and 1 left

In this case, the 1 that is left is also divided by 2, and $1 \div 2 = \frac{1}{2}$. Then, instead of writing 1 left, write $\frac{1}{2}$ in the answer in this way:

$$\begin{array}{r} 2\frac{1}{2} \\ 2\overline{)5} \\ \underline{4} \\ 1 \end{array}$$

$1 \div 2 = \frac{1}{2}$

In division problems where the remainder has to be divided, the remainder is expressed as a fraction.

Mr. Jones bought two copies of a book to give as Christmas gifts. He paid $7 for the two books. How much did each book cost?

$$\begin{array}{r} 3\frac{1}{2} \\ 2\overline{)7} \\ \underline{6} \\ 1 \end{array}$$ $1 \div 2 = \frac{1}{2}$

Each book cost $\$3\frac{1}{2}$ or $3.50.

Divide, and express the remainder as a fraction.

$2\overline{)9}$ $\square \div 2 = \frac{\square}{\square}$

$3\overline{)8}$ $\square \div 3 = \frac{\square}{\square}$

$4\overline{)10}$ $\square \div 4 = \frac{\square}{\square}$

$5\overline{)8}$ $\square \div 5 = \frac{\square}{\square}$

$4\overline{)17}$ $\square \div 4 = \frac{\square}{\square}$

$3\overline{)23}$ $\square \div 3 = \frac{\square}{\square}$

$10 \times 7 = \square$.
$20 \times 7 = \square$.
$1 \times 7 = \square$.
$2 \times 7 = \square$.

$7\overline{)85}$

Answer: $\square\frac{\square}{\square}$

$\square \div 7 = \frac{\square}{\square}$.

$10 \times 8 = \square$.
$20 \times 8 = \square$.
$1 \times 8 = \square$.
$2 \times 8 = \square$.

$8\overline{)90}$

Answer: $\square\frac{\square}{\square}$

$\square \div 8 = \frac{\square}{\square}$.

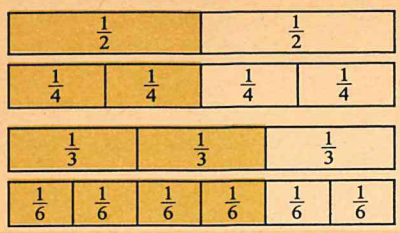
$10 \times 4 = \square$.
$20 \times 4 = \square$.
$30 \times 4 = \square$.
$1 \times 4 = \square$.
$2 \times 4 = \square$.

$4\overline{)86}$

Answer: $\square\frac{\square}{\square}$

$\square \div 4 = \frac{\square}{\square}$.

Changing to Higher Terms



$\frac{1}{2} = \frac{2}{4} = \frac{2 \times 1}{2 \times 2}$.

$\frac{2}{3} = \frac{4}{6} = \frac{2 \times 2}{2 \times 3}$.

To change a fraction to higher terms, multiply both numerator and denominator by the same number.

Write the missing numbers to change each fraction to higher terms.

$\frac{1}{5} = \frac{\square \times 1}{2 \times 5} = \frac{\square}{\square}$.

$\frac{1}{6} = \frac{2 \times 1}{\square \times 6} = \frac{\square}{\square}$.

$\frac{1}{3} = \frac{\square \times 1}{3 \times 3} = \frac{\square}{\square}$.

$\frac{3}{5} = \frac{\square \times 3}{3 \times 5} = \frac{\square}{\square}$.

$\frac{2}{7} = \frac{3 \times 2}{\square \times 7} = \frac{\square}{\square}$.

$\frac{1}{2} = \frac{\square \times 1}{3 \times 2} = \frac{\square}{\square}$.

$\frac{1}{10} = \frac{5 \times 1}{\square \times 10} = \frac{\square}{\square}$.

$\frac{1}{3} = \frac{\square \times 1}{4 \times 3} = \frac{\square}{\square}$.

$\frac{2}{4} = \frac{2 \times 2}{\square \times 4} = \frac{\square}{\square}$.

$\frac{3}{10} = \frac{10 \times 3}{\square \times 10} = \frac{\square}{\square}$.

$\frac{2}{10} = \frac{10 \times 2}{\square \times 10} = \frac{\square}{\square}$.

$\frac{1}{2} = \frac{5 \times 1}{\square \times 2} = \frac{\square}{\square}$.

$\frac{1}{4} = \frac{\square \times 1}{2 \times 4} = \frac{\square}{\square}$.

$\frac{2}{3} = \frac{\square \times 2}{3 \times 3} = \frac{\square}{\square}$.

$\frac{1}{2} = \frac{\square \times 1}{\square \times 2} = \frac{\square}{6}$.

$\frac{1}{2} = \frac{\square \times 1}{\square \times 2} = \frac{\square}{4}$.

$\frac{1}{2} = \frac{\square \times 1}{\square \times 2} = \frac{\square}{10}$.

$\frac{1}{2} = \frac{\square \times 1}{\square \times 2} = \frac{\square}{8}$.

$\frac{1}{3} = \frac{\square \times 1}{\square \times 3} = \frac{\square}{6}$.

$\frac{1}{3} = \frac{\square \times 1}{\square \times 3} = \frac{\square}{9}$.

$\frac{1}{3} = \frac{\square \times 1}{\square \times 3} = \frac{\square}{12}$.

$\frac{1}{4} = \frac{\square \times 1}{\square \times 4} = \frac{\square}{8}$.

$\frac{1}{4} = \frac{\square \times 1}{\square \times 4} = \frac{\square}{12}$.

$\frac{1}{5} = \frac{\square \times 1}{\square \times 5} = \frac{\square}{10}$.

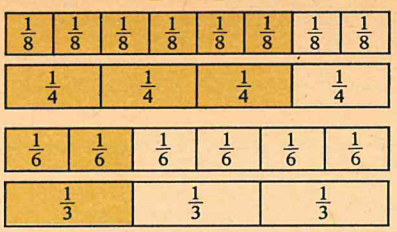
$\frac{1}{5} = \frac{\square \times 1}{\square \times 5} = \frac{\square}{20}$.

$\frac{2}{5} = \frac{\square \times 2}{\square \times 5} = \frac{\square}{10}$.

$\frac{2}{5} = \frac{\square \times 2}{\square \times 5} = \frac{\square}{15}$.

$\frac{3}{5} = \frac{\square \times 3}{\square \times 5} = \frac{\square}{15}$.

Changing to Lower Terms



$\frac{6}{8} = \frac{3}{4} = \frac{6 \div 2}{8 \div 2}$.

$\frac{2}{6} = \frac{1}{3} = \frac{2 \div 2}{6 \div 2}$.

To change a fraction to lower terms, divide both numerator and denominator by the same number.

Write the missing numbers to change each fraction to lower terms.

$\frac{4}{6} = \frac{4 \div \square}{6 \div 2} = \frac{\square}{\square}$.

$\frac{2}{4} = \frac{2 \div \square}{4 \div 2} = \frac{\square}{\square}$.

$\frac{4}{8} = \frac{4 \div 4}{8 \div \square} = \frac{\square}{\square}$.

$\frac{4}{8} = \frac{4 \div 2}{8 \div \square} = \frac{\square}{\square}$.

$\frac{6}{9} = \frac{6 \div \square}{9 \div 3} = \frac{\square}{\square}$.

$\frac{5}{10} = \frac{5 \div 5}{10 \div \square} = \frac{\square}{\square}$.

$\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{\square}{\square}$.

$\frac{8}{10} = \frac{8 \div 2}{10 \div \square} = \frac{\square}{\square}$.

$\frac{6}{12} = \frac{6 \div 6}{12 \div \square} = \frac{\square}{\square}$.

$\frac{6}{12} = \frac{6 \div 3}{12 \div \square} = \frac{\square}{\square}$.

$\frac{4}{10} = \frac{4 \div \square}{10 \div 2} = \frac{\square}{\square}$.

$\frac{2}{8} = \frac{2 \div \square}{8 \div 2} = \frac{\square}{\square}$.

$\frac{4}{16} = \frac{4 \div 4}{16 \div \square} = \frac{\square}{\square}$.

$\frac{8}{16} = \frac{8 \div 8}{16 \div \square} = \frac{\square}{\square}$.

$\frac{3}{12} = \frac{3 \div \square}{12 \div \square} = \frac{\square}{4}$.

$\frac{2}{4} = \frac{2 \div \square}{4 \div \square} = \frac{\square}{2}$.

$\frac{4}{10} = \frac{4 \div \square}{10 \div \square} = \frac{\square}{5}$.

$\frac{6}{12} = \frac{6 \div \square}{12 \div \square} = \frac{\square}{2}$.

$\frac{3}{15} = \frac{3 \div \square}{15 \div \square} = \frac{\square}{5}$.

$\frac{6}{10} = \frac{6 \div \square}{10 \div \square} = \frac{\square}{5}$.

$\frac{8}{12} = \frac{8 \div \square}{12 \div \square} = \frac{\square}{3}$.

$\frac{6}{9} = \frac{6 \div \square}{9 \div \square} = \frac{\square}{3}$.

$\frac{2}{10} = \frac{2 \div \square}{10 \div \square} = \frac{\square}{5}$.

$\frac{10}{15} = \frac{10 \div \square}{15 \div \square} = \frac{\square}{3}$.

$\frac{4}{12} = \frac{4 \div \square}{12 \div \square} = \frac{\square}{3}$.

$\frac{8}{10} = \frac{8 \div \square}{10 \div \square} = \frac{\square}{5}$.

$\frac{6}{8} = \frac{6 \div \square}{8 \div \square} = \frac{\square}{4}$.

$\frac{5}{15} = \frac{5 \div \square}{15 \div \square} = \frac{\square}{3}$.

Comparing Units, Tens, Hundreds, and Thousands

Unit means one. The ones place in a number is also called the units place.

Thousands	Hundreds	Tens	Units	
				$10 \times 1 = 10$. 1 ten is 10 times 1 unit. $10 \times 10 = 100$. 1 hundred is 10 times 1 ten. $10 \times 100 = 1000$. 1 thousand is 10 times 1 hundred.
			1	This 1 stands for 1 unit.
		1	0	This 1 stands for 1 ten, which is 10 times 1 unit.
		1 ↙	1 0	If 1 moves from the units place to the tens place, it becomes worth 10 times as much.
		1	0	This 1 stands for 1 ten.
	1	0	0	This 1 stands for 1 hundred, which is ☐ times 1 ten.
	1 ↙	1 0	0	If 1 moves from the tens place to the hundreds place, it becomes worth ☐ times as much.
	1	0	0	This 1 stands for 1 hundred.
1	0	0	0	This 1 stands for 1 thousand, which is ☐ times 1 hundred.
1 ↙	1 0	0	0	If 1 moves from the hundreds place to the thousands place, it becomes worth ☐ times as much.
		2	0	2 tens $= 20 = 10 \times 2$. 2 tens $=$ ☐ $\times$ 2 units.
	2	0	0	2 hundreds $= 200 = 10 \times 20$. 2 hundreds $=$ ☐ $\times$ 2 tens.
2	0	0	0	2 thousands $= 2000 = 10 \times 200$. 2 thousands $=$ ☐ $\times$ 2 hundreds.
		3	0	3 tens $= 30 = 10 \times 3$. 3 tens $=$ ☐ $\times$ 3 units.
	3	0	0	3 hundreds $= 300 = 10 \times 30$. 3 hundreds $=$ ☐ $\times$ 3 tens.
3	0	0	0	3 thousands $= 3000 = 10 \times 300$. 3 thousands $=$ ☐ $\times$ 3 hundreds.

Thousands	Hundreds	Tens	Units	1 ten $= 1 \times 10 = 10 \times 1$ unit. 1 hundred $= 1 \times 100$ $= 100 \times 1$ unit. 1 thousand $= 1 \times 1000$ $= 1000 \times 1$ unit.
			5	5 tens $= 5 \times 10$ $=$ ☐ $\times$ 5 units.
		5	0	
			5	5 hundreds $= 5 \times 100$ $=$ ☐ $\times$ 5 units.
	5	0	0	
			5	5 thousands $= 5 \times 1000$ $=$ ☐ $\times$ 5 units.
5	0	0	0	
			6	6 tens $= 6 \times 10$ $=$ ☐ $\times$ 6 units.
		6	0	
			6	6 hundreds $= 6 \times 100$ $=$ ☐ $\times$ 6 units.
	6	0	0	
			6	6 thousands $= 6 \times 1000$ $=$ ☐ $\times$ 6 units.
6	0	0	0	
			7	7 tens $= 7 \times$ ☐ $=$ ☐ $\times$ 7 units.
		7	0	
			7	7 hundreds $= 7 \times$ ☐ $=$ ☐ $\times$ 7 units.
	7	0	0	
			7	7 thousands $= 7 \times$ ☐ $=$ ☐ $\times$ 7 units.
7	0	0	0	
			8	8 tens $= 8 \times$ ☐ $=$ ☐ $\times$ 8 units.
		8	0	
			8	8 hundreds $= 8 \times$ ☐ $=$ ☐ $\times$ 8 units.
	8	0	0	
			8	8 thousands $= 8 \times$ ☐ $=$ ☐ $\times$ 8 units.
8	0	0	0	

More Roman Numerals

In Roman numerals, D means 500.

DC = ☐ + 100 = ☐.

CD = ☐ − 100 = ☐.

DL = ☐ + ☐ = ☐.

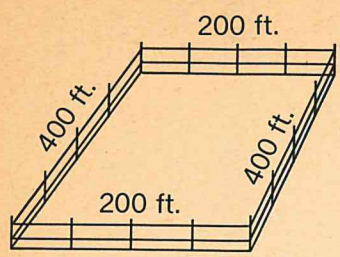
DX = ☐ + ☐ = ☐.

DI = ☐ + ☐ = ☐.

DV = ☐ + ☐ = ☐.

DCC = ☐. DXV = ☐.

Perimeter



Mr. Jones has put a fence around his field. The field is a rectangle that is 400 ft. long and 200 ft. wide. How many feet of fencing did he use?

This question can be answered in 3 ways.

1st Way

400 ft.
200 ft.
400 ft.
+200 ft.
_____
☐ ft.

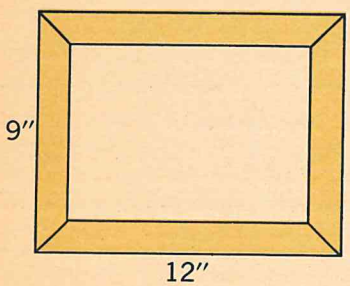
2nd Way

2 × 400 ft. = ☐ ft.
2 × 200 ft. = ☐ ft.
Total = ☐ ft.

3rd Way

400 ft. + 200 ft. = 600 ft.
2 × 600 ft. = ☐ ft.

The distance around a rectangle is called its perimeter.



How many inches of picture-frame molding are needed to make a frame 12 in. long and 9 in. wide?

Find the answer in three ways.

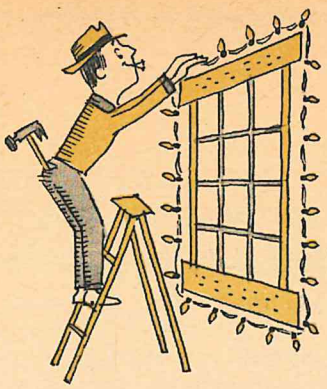
1st Way

2nd Way

3rd Way

What is the perimeter of a square that is 8 ft. long on each side?

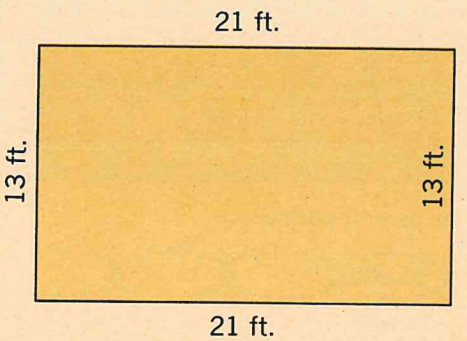
Answer: ☐ ft.



To decorate his window for Christmas, Mr. Smith has put colored lights, strung on wire, around the window. How many feet of wire did he use, if the window frame is 8 ft. long and 4 ft. wide?

Answer: ☐ ft.

Find the perimeter of a rectangle, if its length is 21 ft., and its width is 13 ft. A plan for the rectangle is drawn below.



Answer: ☐ ft.

Draw a plan for each rectangle. Then find its perimeter.

length = 32 yards
width = 15 yards

length = 125 feet
width = 95 feet

Five-Digit Numerals

Ten Thousands	Thousands	Hundreds	Tens	Units	
3	4	1	2	5	= 3 ten thousands + 4 thousands + 1 hundred + 2 tens + 5 units.
4	0	6	2	0	= ☐ ten thousands + ☐ thousands + ☐ hundreds + ☐ tens + ☐ units.

To read the first numeral aloud, say:
thirty-four thousand, one hundred twenty-five.
To read the second numeral, say:
forty thousand, six hundred twenty.

Write each of these numbers in numerals.

fifty-two thousand, three hundred eighteen					
twenty thousand, one hundred six					
fifteen thousand, two hundred					

Write in the rings as you count

by hundreds: 56,800 ○ ○ ○

by thousands: 29,000 ○ ○ ○

by ten thousands: 50,000 ○ ○ ○

1 ten thousand = 10 thousands.

3 ten thousands = ☐ thousands.

5 ten thousands = ☐ thousands.

60 thousands = ☐ ten thousands.

Exchange 10 thousands for 1 ten thousand.

5 ten thousands + 18 thousands
= ☐ ten thousands + 8 thousands.

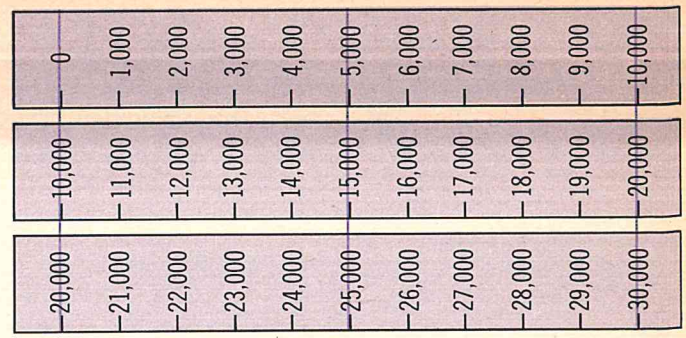
2 ten thousands + 15 thousands
= 3 ten thousands + ☐ thousands.

Exchange 1 ten thousand for 10 thousands.

7 ten thousands + 1 thousand
= 6 ten thousands + ☐ thousands.

5 ten thousands + 0 thousands
= ☐ ten thousands + 10 thousands.

The Nearest Ten Thousand



5,000 is halfway between 0 and 10,000.

15,000 is halfway between 10,000 and ☐.

25,000 is halfway between ☐ and 30,000.

13,000 is nearer to 10,000 than to 20,000.

18,000 is nearer to ☐ than to 10,000.

29,000 is nearer to ☐ than to 20,000.

22,000 is nearer to ☐ than to 30,000.

Round off to the nearest ten thousand.

26,000 is about ☐. 14,000 is about ☐.

58,129 is about ☐. 45,600 is about ☐.

31,108 is about ☐. 52,999 is about ☐.

Estimate these sums or differences.

43,215 + 29,412 = about 40,000 + 30,000
= about 70,000.

24,519 + 31,112 = about ☐ + ☐
= about ☐.

86,810 − 22,115 = about ☐ − ☐
= about ☐.

78,500 − 39,400 = about ☐ − ☐
= about ☐.

Add.

12,175 + 13,112 = ☐	24,210 + 24,508 = ☐
17,115 + 14,211 = ☐ (1 ☐)	11 thousands = 1 ten thousand + 1 thousand
25,201 + 18,352 = ☐	45,181 + 15,254 = ☐

Subtract.

45,861 − 23,150 = ☐	78,459 − 35,459 = ☐
41,857 − 13,206 = ☐	Exchange 1 ten thousand for 10 thousands.
63,289 − 19,109 = ☐	50,000 − 5,000 = ☐

Multiples of One Thousand or Ten Thousand

$1 \times 1{,}000 = \square$. $1 \times 10{,}000 = \square$.

$2 \times 1{,}000 = \square$. $3 \times 10{,}000 = \square$.

$4 \times 1{,}000 = \square$. $5 \times 10{,}000 = \square$.

$3 \times 2000 = (3 \times 2) \times 1000 = 6 \times 1000 = \square$.

$4 \times 2000 = (4 \times 2) \times 1000 = \square \times 1000 = \square$.

$5 \times 3000 = (5 \times 3) \times 1000 = \square \times 1000 = \square$.

$6 \times 7000 = \square$. $5 \times 6000 = \square$.

$8 \times 9000 = \square$. $8 \times 8000 = \square$.

$3 \times 20{,}000 = (3 \times 20)$ thousands

$= \square$ thousands $= \square$.

$3 \times 30{,}000 = (3 \times 30)$ thousands

$= \square$ thousands $= \square$.

$4 \times 40{,}000 = (4 \times 40)$ thousands

$= \square$ thousands $= \square$.

A One-Digit Multiplier and a Four- or Five-Digit Multiplicand

Multiply $2 \times 36{,}410$.

Estimate: about $2 \times 40{,}000 = \square$.

First Way

$2 \times 0 =$

$2 \times 10 =$

$2 \times 400 =$

$2 \times 6000 =$

$2 \times 30{,}000 =$

$$\begin{array}{r} 36{,}410 \\ \times 2 \\ \hline \square \end{array}$$

Short Way

2×0 ones $= 0$ ones.

2×1 ten $= 2$ tens.

2×4 hundreds $= 8$ hundreds.

2×6 thousands $= 12$ thousands

$=$ 1 ten thousand $+ 2$ thousands.

2×3 ten thousands $= 6$ ten thousands.

6 ten thousands + 1 ten thousand

$= 7$ ten thousands.

$$\begin{array}{r} 36{,}410 \\ \times 2 \\ \hline 72{,}820 \end{array}$$

Multiply by the short way. Then check your answer by multiplying by the first way.

Multiply $3 \times 27{,}152$.

Estimate: about

$3 \times \square$

$= \square$.

Short Way

$$\begin{array}{r} 27{,}152 \\ \times 3 \\ \hline \square \end{array}$$

Check.

$$\begin{array}{r} 27{,}152 \\ \times 3 \\ \hline \square \end{array}$$

$3 \times 2 =$

$3 \times 50 =$

$3 \times 100 =$

$3 \times 7000 =$

$3 \times 20{,}000 =$

Multiply $4 \times 16{,}413$.

Estimate: about

$4 \times \square$

$= \square$.

Short Way

$$\begin{array}{r} 16{,}413 \\ \times 4 \\ \hline \square \end{array}$$

Check.

$$\begin{array}{r} 16{,}413 \\ \times 4 \\ \hline \square \end{array}$$

$4 \times 3 =$

$4 \times 10 =$

$4 \times 400 =$

$4 \times 6000 =$

$4 \times 10{,}000 =$

Multiply $5 \times 2{,}310$.

Estimate: about

$5 \times \square$

$= \square$.

Short Way

$$\begin{array}{r} 2{,}310 \\ \times 5 \\ \hline \square \end{array}$$

Check.

$$\begin{array}{r} 2{,}310 \\ \times 5 \\ \hline \square \end{array}$$

$5 \times 0 =$

$5 \times 10 =$

$5 \times 300 =$

$5 \times 2000 =$

Multiply $6 \times 3{,}491$.

Estimate: about

$6 \times \square$

$= \square$.

Short Way

$$\begin{array}{r} 3{,}491 \\ \times 6 \\ \hline \square \end{array}$$

Check.

$$\begin{array}{r} 3{,}491 \\ \times 6 \\ \hline \square \end{array}$$

$6 \times 1 =$

$6 \times 90 =$

$6 \times 400 =$

$6 \times 3000 =$

A Multiple of Ten Times a Three- or Four-Digit Numeral

	Ten Thousands	Thousands	Hundreds	Tens	Units
10 ×			2	4	3
=		□	□	□	0
10 ×			5	6	1
=		□	□	□	□
10 ×			4	3	9
=		□	□	□	□

	Ten Thousands	Thousands	Hundreds	Tens	Units
10 ×		4	6	7	8
=	□	□	□	□	0
10 ×		8	1	0	6
=	□	□	□	□	□
10 ×		5	0	9	4
=	□	□	□	□	□

231 × 20 — Put down 0. Move left, and multiply by 2.

425 × 20 — Put down 0. Move left, and multiply by 2.

143 × 30 — Put down 0. Move left, and multiply by 3.

135 × 50 — Put down 0. Move left, and multiply by 5.

3124 × 60 = □□□□□□

285 × 40 = □□□□□

516 × 80 = □□□□□

4219 × 70 = □□□□□□

372 × 50 = □□□□□

452 × 90 = □□□□□

6821 × 20 = □□□□□

702 × 30 = □□□□□

900 × 50 = □□□□□

30 × 270 = □□□□.

40 × 164 = □□□□.

50 × 112 = □□□□.

60 × 108 = □□□□.

A Two-Digit Multiplier and a Three- or Four-Digit Multiplicand

Multiply 24 × 135. The multiplier 24 = 20 + 4.

Multiply by 4.	Multiply by 20.	Then add.
135 × 4 = 540	135 × 20 = 2700	540 + 2700 = □
Step I	Step II	Step III

Multiply 32 × 241. The multiplier 32 = 30 + 2.

241 × 2 =	241 × 30 =	+ =

41 × 1672. The multiplier 41 = 40 + 1.

1672 × 1 =	1672 × 40 =	+ =

Short way of arranging your work:

325 × 34 = □□□□□

Step I—Multiply by 4.

Step II—Multiply by 30.

Step III—Add.

Multiply, arranging your work in the short way.

526 × 25 =	407 × 64 =
1234 × 21 =	2108 × 17 =
3001 × 23 =	4220 × 42 =

Multiplying by a Multiple of One Hundred

$100 \times 4 = 4 \times 100 = 400.$

	Hundreds	Tens	Units
100 ×			4
=	4	0	0

Multiplying 4 by 100 changes 4 units to 4 hundreds. The 4 moves 2 places to the left, and zeros hold the units place and the tens place.

$100 \times 32 = 32 \times 100 = 32$ hundreds
$= 30$ hundreds $+ 2$ hundreds
$= 3$ thousands $+ 2$ hundreds $= 3200.$

	Thousands	Hundreds	Tens	Units
100 ×			3	2
=	3	2	0	0

Multiplying a number by 100 moves each digit 2 places to the left, and zeros hold the units place and the tens place.

100 ×			4	6
=				

100 ×			8	5
=				

100 ×			7	2
=				

100 ×			6	9
=				

$200 \times 3 = 2 \times (100 \times 3) = 2 \times 300 = 600.$

Multiplying by 200 is like multiplying first by 100, and then by 2.

3
×200
6 0 0

First put down 2 zeros and move 2 places left.

Then multiply by 2.

4
×300

5
×400

7
×500

15
×600

21
×300

25
×200

41
×200

52
×300

65
×400

Multiplying by a Multiple of One Thousand

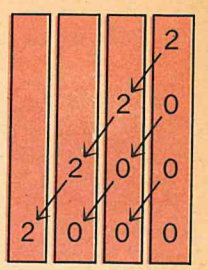
$10 \times 10 = 100.$ $10 \times 10 \times 10 = 1000.$

$1000 \times 2 = (10 \times 10 \times 10) \times 2.$

Multiply 2 by 10:

Multiply 20 by 10:

Multiply 200 by 10:



So $1000 \times 2 = 2000.$

To multiply a number by 1000, move each digit 3 places to the left. Zeros hold the units place, the tens place, and the hundreds place.

1000 ×				5
=				

1000 ×			2	6
=				

1000 ×				7
=				

1000 ×			4	9
=				

$3000 \times 2 = 3 \times (1000 \times 2) = 3 \times 2000 = 6000.$

Multiplying by 3000 is like multiplying first by 1000, and then by 3.

2
×3000
6 0 0 0

First put down 3 zeros and move 3 places left.

Then multiply by 3.

5
×4000

6
×2000

7
×6000

8
×5000

16
×2000

24
×3000

42
×5000

64
×4000

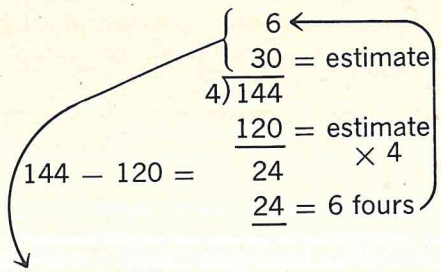
A One-Digit Divisor and a Quotient Less than One Hundred

How many fours are there in 144?

$4 \times 10 = 40$ $\quad 4 \times 20 = 80$

$4 \times 30 = 120$ $\quad 4 \times 40 = 160$

Estimate: over 30



Answer: 36

Check. If the answer is correct, 36×4 should equal 144. By the commutative law of multiplication, $36 \times 4 = 4 \times 36$.

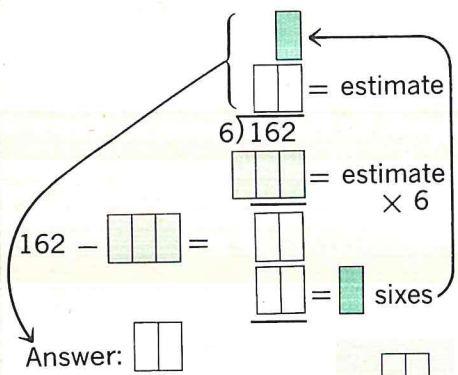
36×4 = ☐☐☐

How many sixes are there in 162?

$6 \times 10 = \square$ $\quad 6 \times 20 = \square$

$6 \times 30 = \square$

Estimate: over ☐



Answer: ☐☐

To check, multiply ☐☐ by ☐.

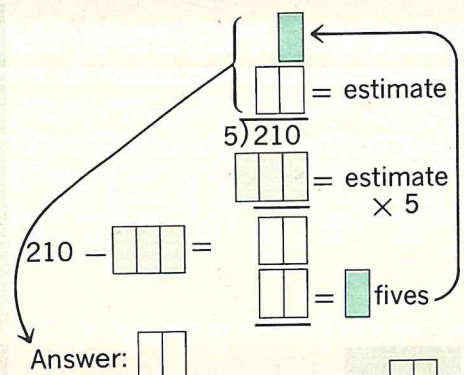
☐☐ × ☐ = ☐☐☐

How many fives are there in 210?

$5 \times 10 = \square$ $\quad 5 \times 20 = \square$

$5 \times 30 = \square$ $\quad 5 \times 40 = \square$

Estimate: over ☐



Answer: ☐☐

To check, multiply ☐☐ by ☐.

☐☐ × ☐ = ☐☐☐

A One-Digit Divisor and a Quotient Less than One Thousand

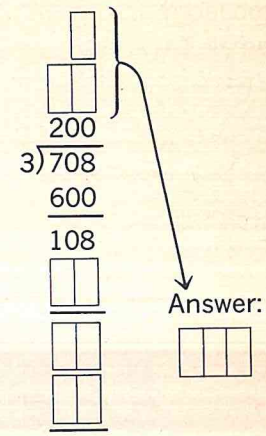
How many threes are there in 708?

$3 \times 100 = 300$ $\quad 3 \times 200 = 600$

$3 \times 300 = 900$ Estimate: over 200

$200 \times 3 = 600$

$708 - 600 = 108$



Now find out how many threes there are in 108:

$3 \times 10 = \square$ $\quad 3 \times 20 = \square$

$3 \times 30 = \square$ $\quad 3 \times 40 = \square$

Check.

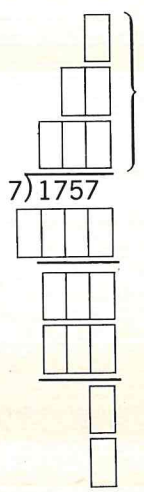
How many sevens are there in 1757?

$7 \times 100 = \square$ $\quad 7 \times 200 = \square$

$7 \times 300 = \square$ Estimate: over ☐

$7 \times 30 = \square$ $\quad 7 \times 40 = \square$

$7 \times 50 = \square$ $\quad 7 \times 60 = \square$



Check.

How many eights are there in 3200?

$8 \times 100 = \square$ $\quad 8 \times 200 = \square$ $\quad 8 \times 300 = \square$ $\quad 8 \times 400 = \square$

Answer: ☐☐☐)3200

A Two-Digit Divisor and a Two-Digit Dividend

Divide 75 by 12.

$10 \times 12 = 120$, which is too large. Try smaller estimates.

12 ×5 ☐ 12 ×6 ☐ 12 ×7 ☐

☐ and ☐ left

12)75

☐

$10 \times 14 =$ ☐ (too large).

14 ×5 ☐ 14 ×6 ☐ 14 ×7 ☐

☐ and ☐ left

14)90

☐

$10 \times 34 =$ ☐ (too large).

34 ×2 ☐ 34 ×3 ☐

☐ and ☐ left

34)70

☐

$10 \times 19 =$ ☐ (too large).

19 ×3 ☐ 19 ×4 ☐ 19 ×5 ☐

☐ and ☐ left

19)88

☐

$10 \times 41 =$ ☐ (too large).

41 ×2 ☐ 41 ×3 ☐

☐ and ☐ left

41)90

☐

25)75

25 ×1 ☐ 25 ×2 ☐ 25 ×3 ☐

18)90

18 ×3 ☐ 18 ×4 ☐ 18 ×5 ☐

Checking Division When There Is a Remainder

7 ×30 ☐ 7 ×40 ☐ 7 ×50 ☐ 7 ×6 ☐ 7 ×7 ☐ 7 ×8 ☐

7)340

Answer: ☐ and ☐ left

First Method of Checking—Add.

Second Method of Checking

☐ (Quotient) × ☐ (Divisor) → ☐ + ☐ (Remainder)

8 ×40 ☐ 8 ×50 ☐ 8 ×60 ☐ 8 ×2 ☐ 8 ×3 ☐ 8 ×4 ☐

8)431

Answer: ☐ and ☐ left

First Check.

Second Check. ☐ (Quotient) × ☐ (Divisor) → ☐ + ☐

9 ×50 ☐ 9 ×60 ☐ 9 ×70 ☐ 9 ×2 ☐

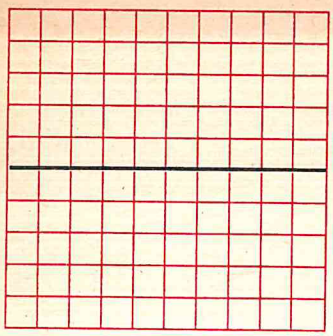
9)563

Answer: ☐ and ☐ left

First Check.

Second Check. ☐ × ☐ → ☐ + ☐

Halves, Fourths, and Eighths Related to Hundredths



The colored lines divide the large square into hundredths.

The black line divides the large square into halves. How many hundredths are there in each half? Count them.

One half = ☐ hundredths; $\frac{1}{2} = \frac{\square}{100}$.

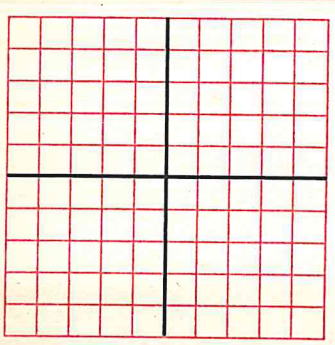
Here is another way to find the answer:

There are 100 hundredths in the whole square. To find $\frac{1}{2}$ of 100, divide 100 by 2. $2\overline{)100}$ ☐

10	20	30	40	50
×2	×2	×2	×2	×2
☐	☐	☐	☐	☐

Here is a third way to find the answer:

Change the fraction $\frac{1}{2}$ to a fraction whose denominator is 100. $\frac{1}{2} = \frac{1 \times \square}{2 \times \square} = \frac{\square}{100}$.



The black lines divide the square into fourths. How many hundredths are there in each fourth? Find the answer in four ways.

First Way

Count the hundredths in a fourth.

One fourth = ☐ hundredths; $\frac{1}{4} = \frac{\square}{100}$.

Second Way

$\frac{1}{4}$ of 100 = 100 ÷ 4.

10 × 4 = ☐.

20 × 4 = ☐.

☐☐ + ☐

= ☐☐

$4\overline{)100}$ ☐☐

☐☐

☐☐

Third Way

Change $\frac{1}{4}$ to higher terms.

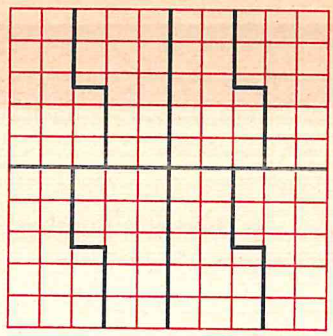
$\frac{1}{4} = \frac{1 \times \square}{4 \times \square} = \frac{\square}{100}$.

Fourth Way

$\frac{1}{4} = \frac{1}{2}$ of $\frac{1}{2}$ =

$\frac{1}{2}$ of 50 hundredths =

☐ hundredths.



The black lines divide the square into eighths. How many hundredths are there in each eighth? Find the answer in three ways.

First Way

Count the hundredths in an eighth.

One eighth = ☐☐ $\frac{1}{2}$ hundredths.

$\frac{1}{8} = \frac{\square\square\ \frac{1}{2}}{100}$.

Second Way

$\frac{1}{8}$ of 100 = 100 ÷ 8.

10 × 8 = 80.

2 × 8 = 16.

4 ÷ 8 =

$\frac{4}{8} = \frac{1}{2}$.

$10 + 2 + \frac{1}{2} = 12\frac{1}{2}$.

$\frac{1}{8} = \frac{12\frac{1}{2}}{100}$.

$8\overline{)100}$ 2, 10, 80, 20, 16, 4

Third Way

$\frac{1}{8} = \frac{1}{2}$ of $\frac{1}{4}$

$= \frac{1}{2}$ of 25 hundredths.

$\frac{1}{2}$ of 25 = 25 ÷ 2.

$2\overline{)25}$ ☐☐, ☐☐, ☐, ☐

Answer: ☐☐ $\frac{\square}{2}$

☐ ÷ ☐ = $\frac{\square}{2}$.

1 fourth = 25 hundredths.

25 + 25 = ☐

2 fourths = ☐ hundredths.

$\frac{2}{4} = \frac{\square}{100}$.

25, 25, 25 + 25 = ☐

3 fourths = ☐ hundredths.

$\frac{3}{4} = \frac{\square}{100}$.

25, 25, 25, 25 + 25 = ☐

4 fourths = ☐ hundredths.

$\frac{4}{4} = \frac{\square}{100}$.

$\frac{2}{4} = \frac{1}{2} = \frac{\square}{100}$.

$\frac{4}{4} = 1 = \frac{\square}{100}$.

Color $\frac{3}{8}$ of the square above. Count the number of hundredths in $\frac{3}{8}$.

$\frac{3}{8} = \frac{\square\ \frac{1}{2}}{100}$.

Color more parts of the square until $\frac{5}{8}$ of the square is colored. Count the number of hundredths in $\frac{5}{8}$.

$\frac{5}{8} = \frac{\square\ \frac{1}{2}}{100}$.

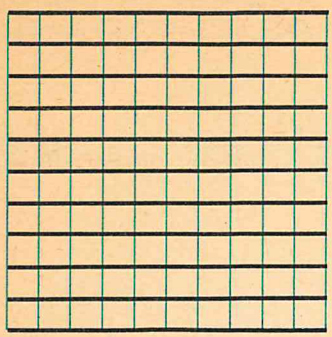
Color more parts of the square until $\frac{7}{8}$ of the square is colored. Count the number of hundredths in $\frac{7}{8}$.

$\frac{7}{8} = \frac{\square\ \frac{1}{2}}{100}$.

$\frac{4}{8} = \frac{1}{2} = \frac{\square}{100}$.

$\frac{6}{8} = \frac{3}{4} = \frac{\square}{100}$.

Tenths and Fifths Related to Hundredths



The black lines divide the square into tenths.

1 tenth = ☐ hundredths.

To find the number of hundredths in 2 tenths, 3 tenths, 4 tenths, and so on, count the hundredths in groups of ten.

2 tenths = ☐ hundredths. $\frac{2}{10} = \frac{\square}{100}$.

3 tenths = ☐ hundredths. $\frac{3}{10} = \frac{\square}{100}$.

4 tenths = ☐ hundredths. $\frac{4}{10} = \frac{\square}{100}$.

5 tenths = ☐ hundredths. $\frac{5}{10} = \frac{\square}{100}$.

6 tenths = ☐ hundredths. $\frac{6}{10} = \frac{\square}{100}$.

7 tenths = ☐ hundredths. $\frac{7}{10} = \frac{\square}{100}$.

8 tenths = ☐ hundredths. $\frac{8}{10} = \frac{\square}{100}$.

9 tenths = ☐ hundredths. $\frac{9}{10} = \frac{\square}{100}$.

10 tenths = ☐ hundredths. $\frac{10}{10} = \frac{\square}{100}$.

Change each fraction below to tenths. Then change it to hundredths.

$\frac{1}{5} = \frac{2 \times 1}{2 \times 5} = \frac{2}{10} = \frac{\square}{100}$.

$\frac{2}{5} = \frac{\square \times 2}{\square \times 5} = \frac{\square}{10} = \frac{\square}{100}$.

$\frac{3}{5} = \frac{\square \times 3}{\square \times 5} = \frac{\square}{10} = \frac{\square}{100}$.

$\frac{4}{5} = \frac{\square \times 4}{\square \times 5} = \frac{\square}{10} = \frac{\square}{100}$.

$\frac{5}{5} = \frac{\square \times 5}{\square \times 5} = \frac{\square}{10} = \frac{\square}{100}$.

Change each fraction to hundredths.

$\frac{1}{8} = \frac{\square}{100}$. $\frac{1}{4} = \frac{\square}{100}$. $\frac{1}{10} = \frac{\square}{100}$.

$\frac{1}{2} = \frac{\square}{100}$. $\frac{3}{4} = \frac{\square}{100}$. $\frac{1}{5} = \frac{\square}{100}$.

Thirds Related to Hundredths

$\frac{1}{3}$ of a whole = $\frac{1}{3}$ of 100 hundredths.

$3 \times 10 = \square$. $3 \times 20 = \square$. $3 \times 30 = \square$.

$1 \div 3 = \frac{1}{3}$.

$$3\overline{)100}\quad 33$$
$$90$$
$$10$$
$$9$$
$$1$$

$\frac{1}{3}$ of 100 = $100 \div 3 = 33\frac{1}{3}$.

$\frac{1}{3} = 33\frac{1}{3}$ hundredths $= \frac{33\frac{1}{3}}{100}$.

$33\frac{1}{3} + 33\frac{1}{3} =$
$33 + 33$
$+ 1 \text{ third} + 1 \text{ third}$
$= \square + \square \text{ thirds}$
$= \square \frac{\square}{3}$.

So $\frac{2}{3} = \frac{\square \frac{\square}{3}}{100}$.

Count by tens. 10 ☐ ☐ ☐

☐ ☐ ☐ ☐ ☐ ☐

Change each fraction to hundredths.

$\frac{3}{10} = \frac{\square}{100}$. $\frac{7}{10} = \frac{\square}{100}$.

$\frac{9}{10} = \frac{\square}{100}$. $\frac{6}{10} = \frac{\square}{100}$.

Count by twenties.

☐ ☐ ☐ ☐ ☐

Change each fraction to hundredths.

$\frac{2}{5} = \frac{\square}{100}$. $\frac{3}{5} = \frac{\square}{100}$.

Circle the correct answer.

$\frac{25}{100} = \frac{1}{2}$ or $\frac{1}{4}$ or $\frac{1}{8}$.

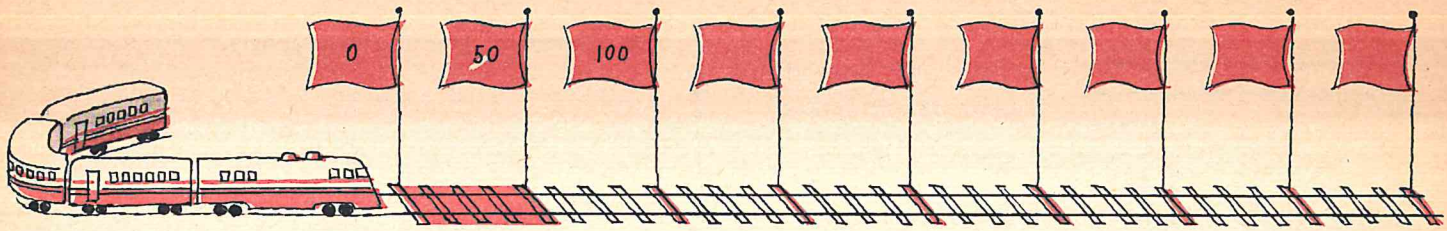
$\frac{40}{100} = \frac{2}{5}$ or $\frac{3}{5}$ or $\frac{1}{5}$.

$\frac{66\frac{2}{3}}{100} = \frac{1}{3}$ or $\frac{2}{3}$ or $\frac{1}{4}$.

$\frac{50}{100} = \frac{1}{4}$ or $\frac{1}{3}$ or $\frac{1}{2}$.

$\frac{60}{100} = \frac{4}{10}$ or $\frac{6}{10}$ or $\frac{8}{10}$.

Rate, Time, and Distance



The colored lines on the track divide it into lengths of 50 miles each. There is a flag at each line. Write on each flag the number of miles it is from the left end of the track.

The train on the track travels at a rate of 50 miles per hour. The distance it travels the first hour is colored. Color the distance the train travels during the second hour. How far does the train travel in 2 hours?

Answer: ☐ miles

Color the distance the train travels during the third hour. How far does the train travel in 3 hours?

Answer: ☐ miles

Color the distance the train travels during the fourth hour. How far does the train travel in 4 hours?

Answer: ☐ miles

Continue in the same way. At each step color the distance the train travels in the next hour, and complete this statement:

The train travels ☐ miles in 5 hours,
☐ miles in 6 hours,
☐ miles in 7 hours,
and ☐ miles in 8 hours.

If a train travels at a rate of 60 miles an hour, how far does it travel in 3 hours?

Answer: 60 miles + 60 miles + 60 miles
= 3 × 60 miles = ☐ miles.

If a train travels at a rate of 70 miles an hour, how far does it travel in 2 hours?

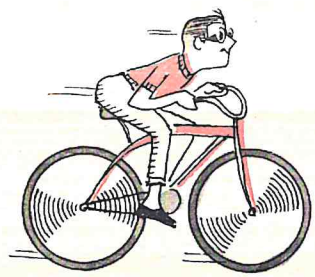
Answer: 70 miles + 70 miles
= 2 × 70 miles = ☐ miles.

Rule for calculating the distance:

Time × Rate = Distance.

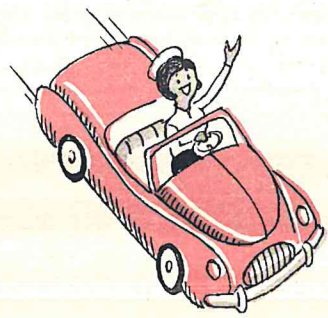
How far can this hiker walk in 3 hours if he walks at a rate of 3 miles per hour?

Answer: ☐



A cyclist kept up a steady speed of 10 miles an hour. How far did he travel in 4 hours?

Answer: ☐



An automobile traveled for 6 hours at a rate of 45 miles an hour. How far did it travel?

Answer: ☐

An airplane traveled for 5 hours at a speed of 340 miles an hour. How far did it travel?

Answer: ☐

Changing Improper Fractions to Mixed or Whole Numbers

A fraction is a proper fraction if the numerator is less than the denominator.

$\frac{1}{2}, \frac{3}{5}, \frac{4}{9}, \frac{10}{12}$ are proper fractions.

$\frac{2}{2}, \frac{3}{2}, \frac{3}{3}, \frac{4}{3}, \frac{9}{5}$ are not proper fractions.

They are called improper fractions.

Put in numerators to make four different proper fractions:

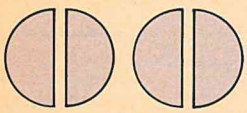
$\frac{\square}{5}, \frac{\square}{5}, \frac{\square}{5}, \frac{\square}{5}$.

The sum of a whole number and a proper fraction is called a mixed number.

Examples of mixed numbers: $2\frac{1}{2}$, $4\frac{2}{3}$, $5\frac{2}{7}$.

Change $\frac{4}{2}$ to a whole number.

First Way Draw 4 halves of a disk.



Each group of 2 halves = 1 whole.

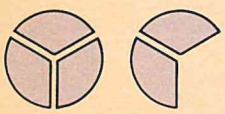
$\frac{4}{2}$ = 2 wholes. $\frac{4}{2} = 2$.

Second Way $\frac{4}{2}$ means $4 \div 2$

$2\overline{)4}$ $\quad 2$

Change $\frac{5}{3}$ to a mixed number.

First Way Draw 5 thirds of a disk.



Each group of 3 thirds = 1 whole.

5 thirds = 1 whole and 2 thirds.

$\frac{5}{3} = 1\frac{2}{3}$.

Second Way $\frac{5}{3}$ means $5 \div 3$

$3\overline{)5}$ $\quad 1\frac{2}{3}$

$\frac{3}{2}$

Change to a mixed or whole number.

$\frac{3}{2} = 3 \div 2 =$

$\frac{5}{2} = \square \div \square =$

$\frac{6}{2} = \square \div \square =$

$\frac{7}{2} = \square \div \square =$

$\frac{8}{2} = \square \div \square =$

$\frac{4}{4} = 4 \div 4 =$

$\frac{5}{4} = \square \div \square =$

$\frac{6}{4} = \square \div \square =$

$\frac{7}{4} = \square \div \square =$

$\frac{8}{4} = \square \div \square =$

Change to a mixed or whole number.

$\frac{8}{3} = \square \div \square =$

$\frac{10}{5} = \square \div \square =$

$\frac{9}{2} = \square \div \square =$

$\frac{11}{8} = \square \div \square =$

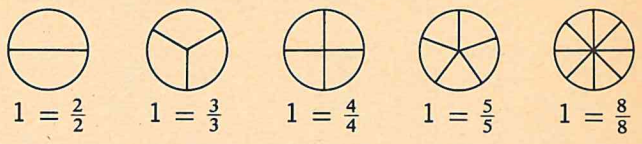
$\frac{6}{5} = \square \div \square =$

$\frac{12}{4} = \square \div \square =$

$\frac{6}{3} = \square \div \square =$

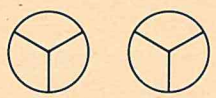
$\frac{7}{3} = \square \div \square =$

Each diagram below shows a way of changing 1 to an improper fraction.



$1 = \frac{2}{2}$ $\quad 1 = \frac{3}{3}$ $\quad 1 = \frac{4}{4}$ $\quad 1 = \frac{5}{5}$ $\quad 1 = \frac{8}{8}$

Change 2 into thirds.



1 = 3 thirds = $\frac{3}{3}$.

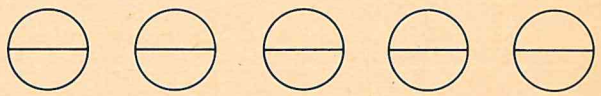
2 = 2 × (3 thirds) = $\frac{6}{3}$.

Change 3 into fourths.

1 = 4 fourths = $\frac{4}{4}$.

3 = $\square$ × (4 fourths) = $\frac{\square}{4}$.

Change 5 into halves.



1 = 2 halves = $\frac{2}{2}$. $\qquad$ 5 = $\square$ × (2 halves) = $\frac{\square}{2}$.

$4 = \frac{\square}{2}$. $\quad 5 = \frac{\square}{3}$. $\quad 6 = \frac{\square}{4}$. $\quad 3 = \frac{\square}{6}$.

Change $2\frac{1}{2}$ into an improper fraction.

First change 2 into halves. 2 = 4 halves.

$2\frac{1}{2}$ = 4 halves + 1 half = 5 halves = $\frac{5}{2}$.

Change $4\frac{1}{3}$ into an improper fraction.

First change 4 into thirds. 4 = $\square$ thirds.

$4\frac{1}{3} = \square$ thirds + 1 third = $\square$ thirds = $\frac{\square}{3}$.

Change into improper fractions.

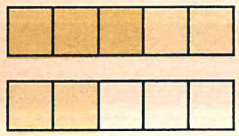
$5\frac{1}{2} = \frac{\square}{2} + \frac{1}{2} = \frac{\square}{2}$.

$4\frac{2}{3} = \frac{\square}{3} + \frac{2}{3} = \frac{\square}{3}$.

$3\frac{1}{4} = \frac{\square}{4} + \frac{1}{4} = \frac{\square}{4}$.

$2\frac{2}{5} = \frac{\square}{5} + \frac{2}{5} = \frac{\square}{5}$.

Addition of Like Fractions



3 fifths

+ 4 fifths

7 fifths

$\frac{3}{5} + \frac{4}{5}$

$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$.

To add fractions that have the same denominator, keep that denominator and add the numerators.

Add.

$\frac{2}{3} + \frac{2}{3} = \frac{\square}{3} = \square\frac{1}{3}$.

$\frac{3}{5} + \frac{2}{5} = \frac{\square}{5} = \square$.

$\frac{3}{4} + \frac{3}{4} = \frac{\square}{4} = \square\frac{\square}{4}$.

$4\frac{2}{3} + 5\frac{2}{3}$

$9\frac{4}{3}$

$= 9 + 1\frac{1}{3}$

$= 10\frac{1}{3}$.

$5\frac{1}{2} + 7\frac{1}{2}$

$12\frac{\square}{2}$

$= 12 + \square$

$= \square$.

$6\frac{3}{4} + 2\frac{2}{4}$

$\square\frac{\square}{4}$

$= \square + \frac{\square}{4}$

$= \square\frac{\square}{4}$.

$3\frac{2}{3} + 4\frac{1}{3} + 5\frac{2}{3}$

$= \square + \square\frac{\square}{\square}$

$= \square\frac{\square}{\square}$.

$1\frac{2}{5} + 5\frac{2}{5} + 8\frac{3}{5}$

$= \square + \square\frac{\square}{\square}$

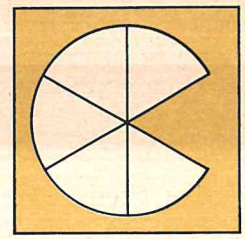
$= \square\frac{\square}{\square}$.

$3\frac{1}{6} + 9\frac{2}{6} + 12\frac{3}{6}$

$\frac{4}{8} + \frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \frac{\square}{\square} = \square$

$\frac{2}{10} + \frac{3}{10} + \frac{5}{10} = \frac{\square}{\square} = \square$

Subtraction of Like Fractions



Fill in 2 more sixths of the disk with crayon.

$\frac{5}{6} - \frac{2}{6} = \frac{\square}{6}$

To subtract fractions that have the same denominators, keep that denominator, and subtract the numerators.

$\frac{4}{3} - \frac{1}{3} = \frac{\square}{3} = \square$.

$\frac{8}{6} - \frac{3}{6} = \frac{\square}{\square}$

$\frac{7}{10} - \frac{2}{10} = \frac{\square}{\square} = \frac{\square}{2}$.

$12\frac{4}{3} - 2\frac{2}{3} = \square\frac{\square}{\square}$

$21\frac{5}{6} - 4\frac{1}{6} = \square\frac{\square}{\square} = \square\frac{\square}{3}$.

$25\frac{7}{8} - 12\frac{3}{8} = \square\frac{\square}{\square} = \square\frac{\square}{2}$.

Subtraction with Exchange.

$15\frac{1}{3} - 8\frac{2}{3} = \square$

Take 1 from 15 and exchange it for 3 thirds.

$15\frac{1}{3} = 14\frac{4}{3}$

$-8\frac{2}{3} = -8\frac{2}{3}$

$\square\frac{\square}{\square}$

$27\frac{1}{5} = 26\frac{\square}{5}$

$-12\frac{3}{5} = -12\frac{3}{5}$

$\square\frac{\square}{\square}$

$37\frac{2}{8} = \square\frac{10}{8}$

$-9\frac{5}{8} = -9\frac{5}{8}$

$\square\frac{\square}{\square}$

$42\frac{1}{6} = \square\frac{\square}{6}$

$-13\frac{5}{6} = -13\frac{5}{6}$

$\square\frac{\square}{\square}$

$29\frac{3}{10} = \square\frac{\square}{10}$

$-15\frac{8}{10} = -15\frac{8}{10}$

$\square\frac{\square}{\square}$

$21\frac{1}{4} = \square\frac{\square}{4}$

$-7\frac{3}{4} = -7\frac{3}{4}$

$\square\frac{\square}{\square}$

$53\frac{1}{8} =$

$-10\frac{7}{8} =$ _____

Decimal Fractions

Th	H	T	U
1	0	0	0
	1	0	0
	1	0	0
		1	0
		1	0
			1

1 thousand = 10 × 1 hundred.

1 hundred = $\frac{1}{10}$ of 1 thousand.

1 hundred = 10 × 1 ten.

1 ten = $\frac{1}{10}$ of 1 hundred.

1 ten = 10 times 1 unit.

1 unit = $\frac{1}{10}$ of 1 ten.

When a digit moves one place to the right, it becomes worth $\frac{1}{10}$ as much.

Th	H	T	U	?
	decimal point		1 .	0
			.	1

Another place has been put in to the right of the units place.

A 1 in this place is worth $\frac{1}{10}$ of 1 unit, or $\frac{1}{10}$. The new place is the tenths place. A decimal point is always put between the units place and the tenths place.

.1 is another way of writing $\frac{1}{10}$.

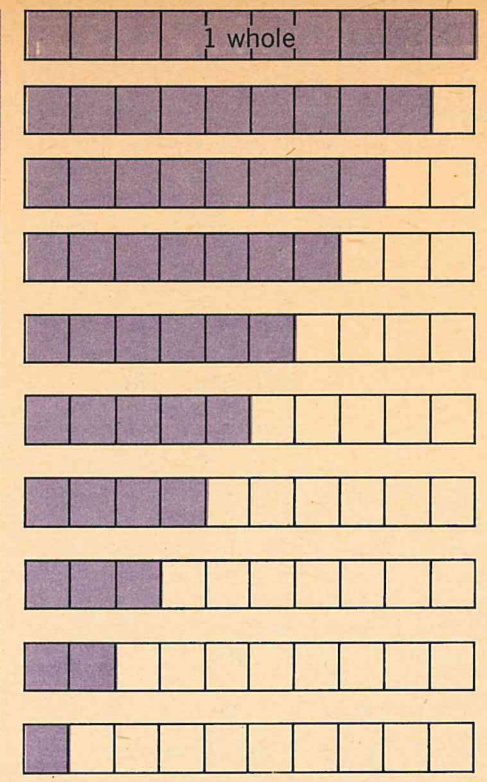
$.1 = \frac{1}{10}$.	$.2 = \frac{2}{10}$.	$.3 = \frac{3}{10}$.
$.4 = \frac{\square}{10}$.	$.5 = \frac{\square}{10}$.	$.6 = \frac{\square}{10}$.
$.7 = \frac{\square}{\square}$.	$.8 = \frac{\square}{\square}$.	$.9 = \frac{\square}{\square}$.

.1, .2, .3, and so on are called decimal fractions or decimals.

Write these numbers as mixed decimals:

	Tens	Units	Tenths
$\frac{4}{10} =$		.	4
$3\frac{2}{10} =$		3 .	2
$5\frac{7}{10} =$		□ .	□
$8\frac{9}{10} =$		□ .	□
$6\frac{1}{10} =$		□ .	□
$12\frac{4}{10} =$	□	□ .	□
$21\frac{5}{10} =$	□	□ .	□
$49\frac{8}{10} =$			
$17\frac{3}{10} =$			
$32\frac{6}{10} =$			

	Tens	Units	Tenths
$24\frac{1}{10} =$	2	4 .	1
$35\frac{7}{10} =$	3	5 .	7
$8\frac{4}{10} =$			
$19\frac{9}{10} =$			
$7\frac{5}{10} =$			
$15\frac{6}{10} =$			
$31\frac{8}{10} =$			
$64\frac{3}{10} =$			
$5\frac{2}{10} =$			
$8 = 8\frac{0}{10} =$			



$1 = \frac{10}{10} = 1.0.$

$\frac{9}{10} = .9.$

$\frac{8}{10} = .8.$

$\frac{7}{10} =$

$\frac{\square}{10} =$

$\frac{\square}{10} =$

$\frac{\square}{10} =$

$\frac{\square}{10} =$

$\frac{\square}{10} =$

$\frac{\square}{10} =$

Count forward by tenths.

.7	.8	.9	1.0	1.1
2.8	2.9	3.0	3.1	3.2
4.8				5.2
6.7				7.1
9.9				10.3

Count backwards by tenths.

1.2	1.1	1.0	.9	.8
5.4				5.0
6.3				5.9
11.0				10.6
13.1				12.7

Count forward .5 at a time.

.5	1.0	1.5	2.0	2.5
3.5				5.5

Count backwards .5 at a time.

7.5	7.0	6.5	6.0	5.5
10.5				8.5
4.0				2.0

Units of Length

1 ft. = 12 in. 1 yd. = 3 ft.

12 ×2 = ☐	12 ×3 = ☐	12 ×4 = ☐	12 ×5 = ☐	12 ×6 = ☐

1 ft. = ☐ in.
2 ft. = ☐ in.
3 ft. = ☐ in.

4 ft. = ☐ in.
5 ft. = ☐ in.
6 ft. = ☐ in.

1 ft. 1 in. = 12 in. + 1 in. = ☐ in.
3 ft. 2 in. = ☐ in. + 2 in. = ☐ in.
4 ft. 6 in. = ☐ in. + 6 in. = ☐ in.
5 ft. 4 in. = ☐ in. + 4 in. = ☐ in.
6 ft. 5 in. = ☐ in. + 5 in. = ☐ in.

15 in. = 12 in. + 3 in. = 1 ft. 3 in.
28 in. = 24 in. + 4 in. = 2 ft. 4 in.
38 in. = ☐ in. + 2 in. = ☐ ft. 2 in.
63 in. = ☐ in. + 3 in. = ☐ ft. 3 in.
50 in. = ☐ in. + ☐ in. = ☐ ft. ☐ in.
75 in. = ☐ in. + ☐ in. = ☐ ft. ☐ in.

Exchange 1 foot for 12 inches.

5 ft. 2 in. = 4 ft. + 12 in. + 2 in. = 4 ft. 14 in.
6 ft. 3 in. = 5 ft. + ☐ in. + 3 in. = 5 ft. ☐ in.
8 ft. 1 in. = ☐ ft. + ☐ in. + 1 in. = ☐ ft. ☐ in.
4 ft. 5 in. = ☐ ft. + ☐ in. + 5 in. = ☐ ft. ☐ in.

2 yd. = (2 × 3) ft. = ☐ ft.
4 yd. = (4 × 3) ft. = ☐ ft.
6 yd. = (6 × 3) ft. = ☐ ft.

3 yd. = ☐ ft.
5 yd. = ☐ ft.
7 yd. = ☐ ft.

6 ft. = 3 ft. + 3 ft. = ☐ yd.
9 ft. = 3 ft. + 3 ft. + 3 ft. = ☐ yd.
12 ft. = 3 ft. + 3 ft. + 3 ft. + 3 ft. = ☐ yd.

1 yd. = 3 ft. = 3 × ☐ in. = ☐ in.

36 ×2 = ☐	36 ×3 = ☐	36 ×4 = ☐	36 ×5 = ☐	36 ×6 = ☐

1 yd. = ☐ in.
2 yd. = ☐ in.
3 yd. = ☐ in.

4 yd. = ☐ in.
5 yd. = ☐ in.
6 yd. = ☐ in.

Units of Weight

1 lb. = 16 oz.
lb. = pounds. oz. = ounces.

16 ×2 = ☐	16 ×3 = ☐	16 ×4 = ☐	16 ×5 = ☐	16 ×6 = ☐

1 lb. = ☐ oz.
2 lb. = ☐ oz.
3 lb. = ☐ oz.

4 lb. = ☐ oz.
5 lb. = ☐ oz.
6 lb. = ☐ oz.

1 lb. 3 oz. = ☐ oz. + 3 oz. = ☐ oz.
2 lb. 4 oz. = ☐ oz. + 4 oz. = ☐ oz.
3 lb. 8 oz. = ☐ oz. + 8 oz. = ☐ oz.

20 oz. = 16 oz. + 4 oz. = 1 lb. 4 oz.
35 oz. = ☐ oz. + 3 oz. = ☐ lb. 3 oz.

Exchange 1 lb. for 16 oz.

5 lb. 1 oz. = 4 lb. + 16 oz. + 1 oz. = 4 lb. ☐ oz.
3 lb. 2 oz. = ☐ lb. + 16 oz. + 2 oz. = ☐ lb. ☐ oz.

Units of Time

1 hr. = 60 min. 1 day = 24 hr.
1 min. = 60 sec. 1 yr. = 12 mo.

60 ×2 = ☐	60 ×3 = ☐	60 ×4 = ☐	60 ×5 = ☐	60 ×6 = ☐

3 hr. = ☐ min.
2 min. = ☐ sec.

4 hr. = ☐ min.
5 min. = ☐ sec.

24 ×2 = ☐	24 ×3 = ☐	24 ×4 = ☐	24 ×5 = ☐	24 ×6 = ☐

3 da. = ☐ hr.
6 da. = ☐ hr.

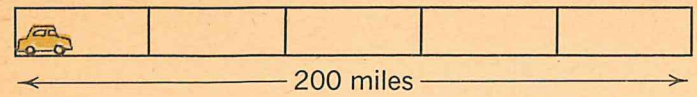
4 da. = ☐ hr.
2 da. = ☐ hr.

90 min. = 60 min. + 30 min. = ☐ hr. 30 min.
130 min. = ☐ min. + 10 min. = ☐ hr. 10 min.

Exchange 1 hour for 60 minutes.

2 hr. 15 min. = 1 hr. + 60 min. + 15 min. = 1 hr. ☐ min.
3 hr. 20 min. = ☐ hr. + 60 min. + 20 min. = ☐ hr. ☐ min.
4 hr. 10 min. = ☐ hr. + 60 min. + 10 min. = ☐ hr. ☐ min.

Finding the Average



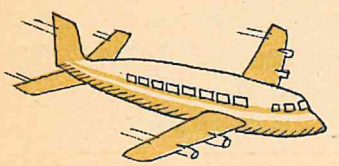
An automobile traveled 200 miles in 5 hours. If it traveled the same number of miles in each hour, how far did it travel in one hour?

Answer: 200 miles ÷ 5 = ☐ miles.

Then the speed of the automobile is ☐ miles per hour (m.p.h.).

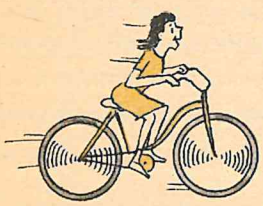
Suppose the automobile traveled 200 miles in 5 hours, but did not travel the same number of miles in each hour. To give a rough idea of how fast the automobile traveled, we make believe that it did travel the same distance in each hour. The speed we calculate in this way is called the average speed of the car.

To find the average speed, divide the total distance by the total time.



An airplane traveled 600 miles in 3 hours. What is its average speed?

Answer: ☐ m.p.h.



A bicycle traveled 24 miles in 2 hours. What is its average speed?

Answer: ☐ m.p.h.

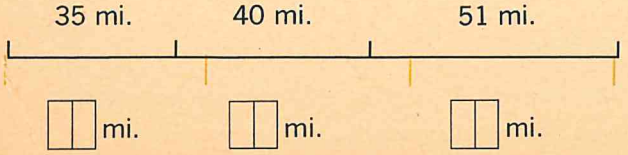
An automobile on a 3-hour trip traveled 35 miles the first hour, 40 miles the second hour, and 51 miles the third hour. First find the total distance. Then divide by the total time.

35 mi.
40 mi.
51 mi.
☐☐☐ mi.

3)☐☐☐

Answer: ☐☐ m.p.h.

Actual distance in each hour—not all equal.



Average distance per hour—all equal.

John took 4 tests in arithmetic. His scores on the tests were 80, 90, 95, 85. His total score was: ☐☐☐.

80
90
95
85
☐☐☐

If his scores were all equal and he had the same total score, each score would have been ☐☐☐ ÷ 4. This is called his average score.

4)☐☐☐

The average score is ☐☐.

To find the average score, add the scores and divide by the number of scores.

Find the average of each set of test scores.

92
94
96
☐☐☐

3)☐☐☐

75
89
82
86
☐☐☐

4)☐☐☐

A class had this attendance record one week. Find the average daily attendance.

M	25
Tu	28
W	27
Th	28
F	27
	☐☐☐

☐)☐☐☐

Roman Numeral for One Thousand

M = 1,000. CM = 1000 − ☐ = ☐.

MM = ☐. MC = 1000 + ☐ = ☐.

MD = 1000 + ☐ = ☐.

MDC = ☐ + ☐ + ☐ = ☐.

MCM = 1000 + ☐ = ☐.

MCML = ☐. MCMLX = ☐.

MCMXX = ☐. MCMXXX = ☐.

MCMXL = ☐. MCMLV = ☐.

MCMLXII = ☐. MCMXLV = ☐.

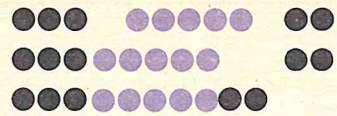
MDCCC = ☐. MDCC = ☐.

MDCCCC = ☐. MDCCLXXVI = ☐.

The Associative Law of Addition

To add three numbers, use two steps.

For example, add $3 + 5 + 2$.



First Way Add $3 + 5$, and then add $8 + 2$.

Write parentheses around $3 + 5$ to show that 3 and 5 were added first.

$(3 + 5) + 2 = 8 + 2 = 10.$

But there is also a second way—Add $5 + 2$, and then add $3 + 7$.

Write parentheses around $5 + 2$ to show that 5 and 2 were added first.

$3 + (5 + 2) = 3 + 7 = 10.$

Since both ways give the same sum, you may write

$(3 + 5) + 2 = 3 + (5 + 2).$

If three numbers are added, you may add the third number to the sum of the first two, or you may add the sum of the second and third to the first number. You get the same sum each way. This is known as the **associative law of addition.**

In each statement below, put in two sets of parentheses to show what the associative law of addition says. Then add in two steps on both sides of the equals sign to see that the sums are really equal.

Example	$8 + 9 + 3 = 8 + 9 + 3.$
Put in parentheses:	$(8 + 9) + 3 = 8 + (9 + 3).$
1st Step:	$17 + 3 = 8 + 12.$
2nd Step:	$20 = 20.$

$5 + 8 + 4 = 5 + 8 + 4.$

Put in parentheses:

1st Step:

2nd Step:

$10 + 3 + 9 = 10 + 3 + 9.$

Put in parentheses:

1st Step:

2nd Step:

The Associative Law of Multiplication

To multiply three numbers, use two steps.

For example, multiply $2 \times 3 \times 4$.

First Way Multiply $2 \times 3 = 6$, and then multiply $6 \times 4 = 24$.

Write parentheses around 2×3 to show that 2 and 3 are multiplied first.

$(2 \times 3) \times 4 = 6 \times 4 = 24.$

There is also a second way. Multiply $3 \times 4 = 12$, and then multiply $2 \times 12 = 24$.

Write parentheses around 3×4 to show that 3 and 4 are multiplied first.

$2 \times (3 \times 4) = 2 \times 12 = 24.$

Since both ways give the same product, you may write

$(2 \times 3) \times 4 = 2 \times (3 \times 4).$

If three numbers are multiplied, you may multiply the third number by the product of the first two, or you may multiply the product of the second and third by the first number. You get the same product each way. This is known as the **associative law of multiplication.**

In each statement below, put in two sets of parentheses to show what the associative law of multiplication says. Then multiply in two steps on both sides of the equals sign to see that the products are really the same.

Example	$2 \times 5 \times 4 = 2 \times 5 \times 4.$
Put in parentheses:	$(2 \times 5) \times 4 = 2 \times (5 \times 4).$
1st Step:	$10 \times 4 = 2 \times 20.$
2nd Step:	$40 = 40.$

$4 \times 3 \times 5 = 4 \times 3 \times 5.$

Put in parentheses:

1st Step:

2nd Step:

$4 \times 5 \times 2 = 4 \times 5 \times 2.$

Put in parentheses:

1st Step:

2nd step:

Six-Digit Numerals

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units	
2	6	1,	3	7	5	= 2 hundred thousands + 6 ten thousands + 1 thousand + 3 hundreds + 7 tens + 5 units.
3	0	0,	1	6	8	= ☐ hundred thousands + ☐ ten thousands + ☐ thousands + ☐ hundred + ☐ tens + ☐ units.

To read the first number above, say: two hundred sixty-one thousand, three hundred seventy-five.

To read the second number, say: three hundred thousand, one hundred sixty-eight.

Write each of these numbers as a numeral:

one hundred twenty-five thousand, two hundred forty-eight	
five hundred thousand, fifteen	

Count

by thousands: 139,000 ☐ ☐ ☐

by ten thousands: 281,000 ☐ ☐ ☐

by hundred thousands: 315,000 ☐ ☐ ☐

1 hundred thousand = 10 ten thousands.

4 hundred thousands = ☐ ten thousands.

7 hundred thousands = ☐ ten thousands.

80 ten thousands = ☐ hundred thousands.

Exchange 10 ten thousands for 1 hundred thousand.

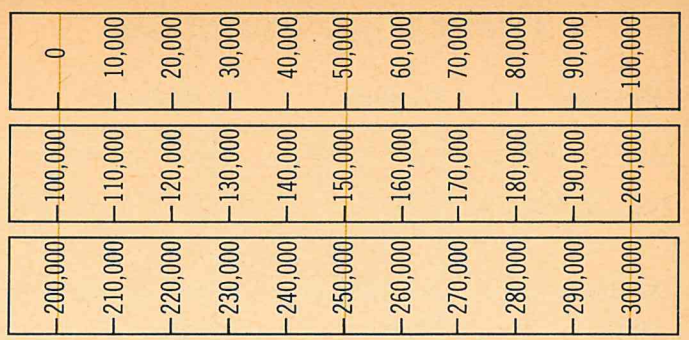
6 hundred thousands + 14 ten thousands
= ☐ hundred thousands + 4 ten thousands.

3 hundred thousands + 17 ten thousands
= 4 hundred thousands + ☐ ten thousands.

Exchange 1 hundred thousand for 10 ten thousands.

5 hundred thousands + 2 ten thousands
= 4 hundred thousands + ☐ ten thousands.

8 hundred thousands + 0 ten thousands
= ☐ hundred thousands + 10 ten thousands.



50,000 is halfway between 0 and 100,000.

150,000 is halfway between 100,000 and ☐.

250,000 is halfway between ☐ and 300,000.

120,000 is nearer to 100,000 than to 200,000.

170,000 is nearer to ☐ than to 100,000.

280,000 is nearer to ☐ than to 200,000.

210,000 is nearer to ☐ than to 300,000.

Round off to the nearest hundred thousand.

360,000 is about ☐. 240,000 is about ☐.

420,000 is about ☐. 290,000 is about ☐.

510,000 is about ☐. 630,000 is about ☐.

Estimate these sums or differences:

261,125 + 392,064 = about 300,000 + 400,000 = about 700,000.

124,189 + 471,870 = about ☐ + ☐ = about ☐.

635,341 − 399,871 = about ☐ − ☐ = about ☐.

Add.

112,415 +231,061 ☐	214,904 +212,062 ☐

Subtract.

682,059 −151,046 ☐	435,257 −104,126 ☐

1
1 5 2,034
+2 8 1,124
☐3☐

13 ten thousands = 1 hundred thousand + 3 ten thousands

612
7 26,459
−153,014

Exchange 1 hundred thousand for 10 ten thousands.

264,127 +192,134 ☐	382,110 +481,010 ☐

538,650 −172,450 ☐	600,000 −70,000 ☐

Three-Digit Multiplier

Multiply
126 × 243.

126 = 100 + 20 + 6.

Multiply by 6.

I
$$\begin{array}{r} 243 \\ \times 6 \\ \hline 1458 \end{array}$$

Multiply by 20.

II
$$\begin{array}{r} 243 \\ \times 20 \\ \hline 4860 \end{array}$$

Multiply by 100.

III
$$\begin{array}{r} 243 \\ \times 100 \\ \hline 24300 \end{array}$$

IV Then add.

$$\begin{array}{r} 1458 \\ 4860 \\ +\,24300 \\ \hline \end{array}$$

Multiply
145 × 387.

145 = 100 + 40 + 5.

Multiply by 5.

I
$$\begin{array}{r} 387 \\ \times 5 \\ \hline \end{array}$$

Multiply by 40.

II
$$\begin{array}{r} 387 \\ \times 40 \\ \hline \end{array}$$

Multiply by 100.

III
$$\begin{array}{r} 387 \\ \times 100 \\ \hline \end{array}$$

IV Then add.

$$+\ \square$$

Multiply
167 × 459.

167 = 100 + 60 + 7.

Multiply by 7.

I
$$\begin{array}{r} 459 \\ \times 7 \\ \hline \end{array}$$

Multiply by 60.

II
$$\begin{array}{r} 459 \\ \times 60 \\ \hline \end{array}$$

Multiply by 100.

III
$$\begin{array}{r} 459 \\ \times 100 \\ \hline \end{array}$$

IV Then add.

$$+\ \square$$

Short Way of Arranging Your Work:

Step I—Multiply by 4.

Step II—Multiply by 80.

Step III—Multiply by 100.

Step IV—Add.

$$\begin{array}{r} 276 \\ \times 184 \\ \hline \square \\ \square \\ \square \\ \hline \square \end{array}$$

Multiply, arranging your work in the short way.

$$\begin{array}{r} 621 \\ \times 138 \\ \hline \end{array}$$

$$\begin{array}{r} 508 \\ \times 159 \\ \hline \end{array}$$

A Two-Digit Divisor and a Three-Digit Dividend

$$\begin{array}{r} 15 \\ \times 10 \\ \hline \square \end{array} \quad \begin{array}{r} 15 \\ \times 20 \\ \hline \square \end{array}$$

$$\begin{array}{r} 15 \\ \times 30 \\ \hline \square \end{array} \quad \begin{array}{r} 15 \\ \times 2 \\ \hline \square \end{array}$$

$$\begin{array}{r} 15 \\ \times 3 \\ \hline \square \end{array} \quad \begin{array}{r} 15 \\ \times 4 \\ \hline \square \end{array}$$

$$15\overline{)360}$$

Answer: $\square$

$$\begin{array}{r} 17 \\ \times 10 \\ \hline \square \end{array} \quad \begin{array}{r} 17 \\ \times 20 \\ \hline \square \end{array}$$

$$\begin{array}{r} 17 \\ \times 30 \\ \hline \square \end{array} \quad \begin{array}{r} 17 \\ \times 40 \\ \hline \square \end{array}$$

$$\begin{array}{r} 17 \\ \times 4 \\ \hline \square \end{array} \quad \begin{array}{r} 17 \\ \times 5 \\ \hline \square \end{array}$$

$$17\overline{)765}$$

Answer: $\square$

$$\begin{array}{r} 26 \\ \times 10 \\ \hline \square \end{array} \quad \begin{array}{r} 26 \\ \times 20 \\ \hline \square \end{array}$$

$$\begin{array}{r} 26 \\ \times 30 \\ \hline \square \end{array} \quad \begin{array}{r} 26 \\ \times 40 \\ \hline \square \end{array}$$

$$\begin{array}{r} 26 \\ \times 1 \\ \hline \square \end{array} \quad \begin{array}{r} 26 \\ \times 2 \\ \hline \square \end{array}$$

$$26\overline{)832}$$

Answer: $\square$

$$\begin{array}{r} 34 \\ \times 10 \\ \hline \square \end{array} \quad \begin{array}{r} 34 \\ \times 20 \\ \hline \square \end{array}$$

$$\begin{array}{r} 34 \\ \times 30 \\ \hline \square \end{array} \quad \begin{array}{r} 34 \\ \times 3 \\ \hline \square \end{array}$$

$$\begin{array}{r} 34 \\ \times 4 \\ \hline \square \end{array} \quad \begin{array}{r} 34 \\ \times 5 \\ \hline \square \end{array}$$

$$34\overline{)852}$$

Answer: $\square$ and $\square$ left

Higher Terms

To change a fraction to higher terms, multiply the numerator and denominator by 2 or 3 or 4 or 5, and so on.

$$\frac{1}{2} = \frac{2 \times 1}{2 \times 2} \text{ or } \frac{3 \times 1}{3 \times 2} \text{ or } \frac{4 \times 1}{4 \times 2} \text{ or } \frac{5 \times 1}{5 \times 2}, \ldots$$

$$= \frac{2}{4} \text{ or } \frac{3}{6} \text{ or } \frac{4}{8} \text{ or } \frac{5}{10}, \ldots$$

The higher denominators are 4, 6, 8, 10, . . . They are the numbers you get when you count by twos.

$$\frac{2}{3} = \frac{2 \times 2}{2 \times 3} \text{ or } \frac{3 \times 2}{3 \times 3} \text{ or } \frac{4 \times 2}{4 \times 3} \text{ or } \frac{5 \times 2}{5 \times 3}, \ldots$$

$$= \frac{4}{6} \text{ or } \frac{6}{9} \text{ or } \frac{8}{12} \text{ or } \frac{10}{15}, \ldots$$

The higher denominators are 6, 9, 12, 15, . . . They are the numbers you get when you count by threes.

$\frac{1}{5}$ may be changed to fractions with denominators 10, ☐, ☐, ☐. To answer, count by 5's.

$$\frac{1}{5} = \frac{\square}{10} \text{ or } \frac{\square}{\square} \text{ or } \frac{\square}{\square} \text{ or } \frac{\square}{\square}.$$

$\frac{3}{4}$ may be changed to fractions with denominators 8, ☐, ☐, ☐. To answer, count by 4's.

$$\frac{3}{4} = \frac{\square}{8} \text{ or } \frac{\square}{\square} \text{ or } \frac{\square}{\square} \text{ or } \frac{\square}{\square}.$$

When you change sixths to higher terms, count by 6's to find the higher denominators.

When you change sevenths to higher terms, count by ☐'s to find the higher denominators.

When you change tenths to higher terms, count by ☐'s to find the higher denominators.

When you change halves to higher terms, count by ☐'s to find the higher denominators.

When you change thirds to higher terms, count by ☐'s to find the higher denominators.

When you change fourths to higher terms, count by ☐'s to find the higher denominators.

Common Denominator in Related Fractions

Change $\frac{1}{2}$ and $\frac{3}{4}$ to fractions that have the same denominator. Counting by 2's you get 2, 4, 6, 8, 10, 12, and so on. Counting by 4's you get 4, 8, 12, 16, and so on.

$\frac{1}{2}$ can be changed to fourths.

$\frac{3}{4}$ cannot be changed to halves.

Use 4 as the **common denominator:**

$$\frac{1}{2} \text{ and } \frac{3}{4} = \frac{2}{4} \text{ and } \frac{3}{4}.$$

Change $\frac{1}{5}$ and $\frac{7}{10}$ to fractions that have the same denominator. 10 can be reached by counting by 5's. Use 10 as the common denominator:

$$\frac{1}{5} \text{ and } \frac{7}{10} = \frac{\square}{10} \text{ and } \frac{\square}{10}.$$

Change $\frac{1}{3}$ and $\frac{1}{6}$ to fractions that have the same denominator. 6 can be reached by counting by 3's. Use 6 as the common denominator:

$$\frac{1}{3} \text{ and } \frac{1}{6} = \frac{\square}{6} \text{ and } \frac{\square}{6}.$$

Change $\frac{1}{2}$ and $\frac{5}{8}$ to fractions that have the same denominator. 8 can be reached by counting by 2's. Use 8 as the common denominator:

$$\frac{1}{2} \text{ and } \frac{5}{8} = \frac{\square}{8} \text{ and } \frac{\square}{8}.$$

Change each pair of fractions into fractions that have the same denominator.

$\frac{1}{3}$ and $\frac{2}{9}$ = ☐/☐ and ☐/☐.

$\frac{1}{4}$ and $\frac{1}{12}$ = ☐/☐ and ☐/☐.

$\frac{1}{2}$ and $\frac{3}{10}$ = ☐/☐ and ☐/☐.

$\frac{2}{3}$ and $\frac{5}{12}$ = ☐/☐ and ☐/☐.

$\frac{3}{4}$ and $\frac{1}{8}$ = ☐/☐ and ☐/☐.

$\frac{2}{7}$ and $\frac{1}{14}$ = ☐/☐ and ☐/☐.

$\frac{1}{6}$ and $\frac{1}{12}$ = ☐/☐ and ☐/☐.

$\frac{3}{10}$ and $\frac{1}{2}$ = ☐/☐ and ☐/☐.

$\frac{1}{4}$ and $\frac{1}{2}$ = ☐/☐ and ☐/☐.

$\frac{2}{15}$ and $\frac{1}{5}$ = ☐/☐ and ☐/☐.

$\frac{4}{15}$ and $\frac{1}{3}$ = ☐/☐ and ☐/☐.

$\frac{7}{10}$ and $\frac{2}{5}$ = ☐/☐ and ☐/☐.

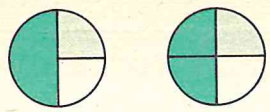
$\frac{1}{6}$ and $\frac{1}{2}$ = ☐/☐ and ☐/☐.

$\frac{7}{12}$ and $\frac{5}{6}$ = ☐/☐ and ☐/☐.

Addition of Unlike, but Related Fractions

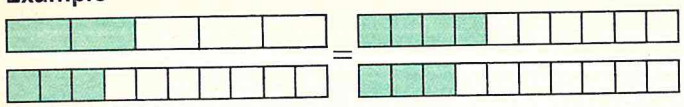
To add unlike fractions, first change them to fractions with a common denominator.

Example



$$\begin{array}{r} \frac{1}{2} = \frac{2}{4} \\ +\frac{1}{4} = \frac{1}{4} \\ \hline \frac{3}{4} \text{ Answer} \end{array}$$

Example



$\frac{2}{5} + \frac{3}{10} = \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$ Answer.

Example $\frac{2}{3} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6} = \frac{9}{6} = 1\frac{3}{6} = 1\frac{1}{2}$.

$\frac{1}{3} = \frac{\square}{6}$

$+\frac{1}{6} = \frac{\square}{6}$

$\frac{\square}{\square} = \frac{\square}{\square}$.

$\frac{1}{5} = \frac{\square}{15}$

$+\frac{4}{15} = \frac{\square}{15}$

$\frac{\square}{15}$

$\frac{1}{4} = \frac{\square}{8}$

$+\frac{1}{8} = \frac{\square}{8}$

$\frac{\square}{8}$

$\frac{1}{9} = \frac{\square}{\square}$

$+\frac{2}{3} = \frac{\square}{\square}$

$\frac{4}{7} = \frac{\square}{\square}$

$+\frac{1}{14} = \frac{\square}{\square}$

$8\frac{3}{5} = 8\frac{\square}{\square}$

$+5\frac{1}{10} = 5\frac{\square}{\square}$

$\frac{5}{8} + \frac{3}{4} = \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$.

$\frac{3}{5} + \frac{7}{10} = \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$.

$\frac{4}{9} + \frac{2}{3} = \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$.

$\frac{5}{6} + \frac{1}{3} = \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$.

$9\frac{1}{2}$
$+4\frac{3}{4}$

$8\frac{1}{2}$
$+12\frac{3}{8}$

Subtraction of Unlike, but Related, Fractions

To subtract unlike fractions, first change them to fractions with a common denominator.

Example

$8\frac{3}{4} = 8\frac{3}{4}$

$-3\frac{1}{2} = -3\frac{2}{4}$

$5\frac{1}{4}$

Example

$9\frac{1}{6} = 9\frac{1}{6} = 8\frac{7}{6}$

$-4\frac{1}{3} = -4\frac{2}{6} = -4\frac{2}{6}$

$4\frac{5}{6}$

Example $\frac{7}{10} - \frac{1}{5} = \frac{7}{10} - \frac{2}{10} = \frac{\square}{\square} = \frac{\square}{\square}$.

$19\frac{1}{5} = 19\frac{\square}{10}$

$-5\frac{1}{10} = -5\frac{\square}{10}$

$\frac{\square}{10}$

$21\frac{5}{8} = \frac{\square}{\square}$

$-7\frac{1}{4} = -\frac{\square}{\square}$

$\frac{\square}{\square}$

$12\frac{7}{10} = \frac{\square}{\square}$

$-3\frac{2}{5} = -\frac{\square}{\square}$

$\frac{\square}{\square}$

$26\frac{7}{8}$
$-2\frac{3}{4}$

$35\frac{5}{12}$
$-4\frac{1}{3}$

$9\frac{1}{4}$
$-3\frac{1}{4}$

$\frac{5}{9} - \frac{1}{3} = \frac{\square}{9} - \frac{\square}{9} = \frac{\square}{9}$.

$\frac{7}{8} - \frac{1}{4} = \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$.

$\frac{5}{6} - \frac{2}{3} = \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$.

$10\frac{1}{4} = \square\frac{1}{4} = \square\frac{5}{4}$

$-3\frac{1}{2} = -\square\frac{\square}{\square} = -\square\frac{\square}{\square}$

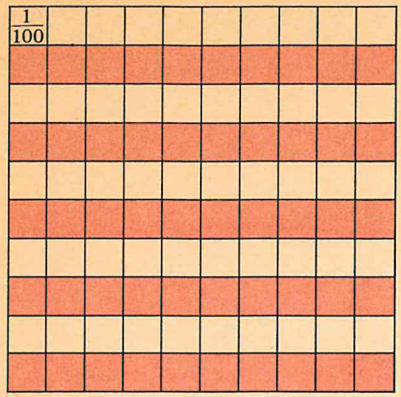
$\square\frac{\square}{\square}$

$17\frac{1}{2}$
$-4\frac{5}{8}$

$25\frac{1}{9}$
$-6\frac{2}{3}$

$18\frac{1}{12}$
$-6\frac{5}{6}$

Decimal Fractions



The large square stands for 1 unit.

It is divided into 100 equal parts. Each part is 1 hundredth ($\frac{1}{100}$).

There are 10 rows of small squares in the whole. Each row is 1 tenth ($\frac{1}{10}$).

There are 100 hundredths in 1 unit.

1 unit is ☐ times as large as 1 hundredth.

1 hundredth is $\frac{1}{100}$ as large as 1 unit.

There are 10 hundredths in 1 tenth.

1 tenth is ☐ times as large as 1 hundredth.

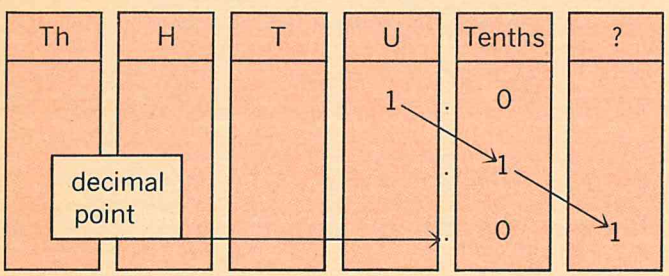
1 hundredth is $\frac{1}{10}$ as large as 1 tenth.

2 tenths = 20 hundredths. 90 hundredths = ☐ tenths.

3 tenths = ☐ hundredths. 80 hundredths = ☐ tenths.

4 tenths = ☐ hundredths. 70 hundredths = ☐ tenths.

When a digit moves 1 place to the right it becomes worth $\frac{1}{10}$ as much.



A 1 in the tenths place is worth $\frac{1}{10}$ of 1 unit. A new place has been put to the right of the tenths place. A 1 in this place is worth $\frac{1}{10}$ of 1 tenth, or 1 hundredth. The new place is the **hundredths** place. The 0 in the tenths place means no tenths.

.01 is another way of writing $\frac{1}{100}$.

.02 is another way of writing $\frac{2}{100}$.

.03 is another way of writing $\frac{\square}{100}$.

.04 is another way of writing $\frac{\square}{100}$.

.05 is another way of writing $\frac{\square}{100}$.

In the number 5.06, the digit in the units place is ☐. The digit in the tenths place is ☐. The digit in the hundredths place is ☐. To read the number, say: Five and six hundredths.

Fill in the missing words.

7.02 = seven and ☐ hundredths.

8.09 = eight and ☐ hundredths.

6.04 = six and four ☐.

7.2 = seven and two ☐.

To write "five and three hundredths" as a decimal, put 5 in the units place, put 0 in the tenths place, and put 3 in the hundredths place.

Five and three hundredths = 5.03.

Write each number as a decimal.

two and four hundredths = ☐.☐☐

three and five hundredths = ☐.☐☐

six and seven hundredths = ☐.☐☐

.12 means 1 tenth and 2 hundredths.
But 1 tenth = 10 hundredths, and
10 hundredths + 2 hundredths = 12 hundredths.
So, to read .12, say 12 hundredths.

.13 = 13 hundredths. .17 = ☐☐ hundredths.

.21 = ☐☐ hundredths. .32 = ☐☐ hundredths.

17.15 = seventeen and 15 hundredths.

27.31 = twenty-seven and ☐☐ hundredths.

Count forward by hundredths.

.01	.02					
						.14
.95				.99	1.00	1.01
1.96			1.99	2.00		
2.96						

Count backwards by hundredths.

2.02	2.01	2.00	1.99			
3.02						
4.01						

Write as a common fraction.

$.07 = \frac{7}{100}$. $.05 = \frac{\square}{\square}$. $.08 = \frac{\square}{\square}$.

$.13 = \frac{\square}{\square}$. $.21 = \frac{\square}{\square}$. $.36 = \frac{\square}{\square}$.

Write as a decimal fraction.

$\frac{6}{100} = \square$. $\frac{4}{100} = \square$. $\frac{9}{100} = \square$.

$\frac{19}{100} = \square$. $\frac{28}{100} = \square$. $\frac{41}{100} = \square$.

$\frac{50}{100} = \square$. $\frac{60}{100} = \square$. $\frac{75}{100} = \square$.

Write as a mixed number.

$3.06 = 3\frac{6}{100}$. $5.12 = \square\frac{\square}{\square}$. $3.05 = \square\frac{\square}{\square}$.

$7.25 = \square\frac{\square}{\square}$. $8.10 = \square\frac{\square}{\square}$. $7.40 = \square\frac{\square}{\square}$.

Write as a mixed decimal.

$10\frac{12}{100} = 10.12$. $5\frac{25}{100} = \square$. $4\frac{10}{100} = \square$.

$23\frac{8}{100} = \square$. $8\frac{31}{100} = \square$. $6\frac{1}{100} = \square$.

Change to hundredths.

$.1 = \frac{1}{10} = \frac{10}{100} = .10$. $.1 = .10$.

$.2 = \frac{2}{10} = \frac{\square}{100} = .\square\square$. $.2 = .\square\square$.

$.3 = \frac{\square}{10} = \frac{\square}{100} = .\square\square$. $.3 = .\square\square$.

$.4 = .\square\square$. $.5 = .\square\square$. $.6 = .\square\square$.

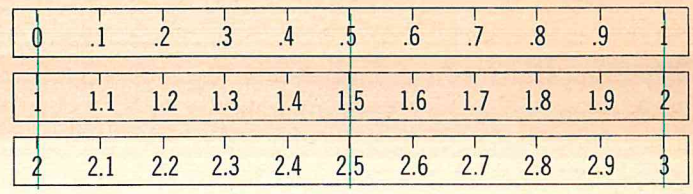
$.7 = .\square\square$. $.8 = .\square\square$. $.9 = .\square\square$.

Change to tenths.

$.20 = \frac{20}{100} = \frac{2}{10} = .2$. $.20 = .2$.

$.50 = \frac{\square}{100} = \frac{\square}{10} = .\square$. $.50 = .\square$.

The Nearest Unit



.5 is halfway between 0 and 1.

1.5 is halfway between 1 and $\square$.

2.5 is halfway between $\square$ and 3.

.3 is nearer to 0 than to 1.

.9 is nearer to $\square$ than to 0.

1.2 is nearer to $\square$ than to 2.

1.7 is nearer to $\square$ than to 1.

2.8 is nearer to $\square$ than to 2.

2.1 is nearer to $\square$ than to 3.

Round off to the nearest unit.

.9 is about 1. .2 is about 0.

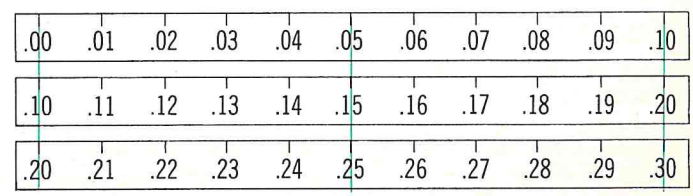
2.1 is about $\square$. 2.6 is about $\square$.

4.8 is about $\square$. 5.7 is about $\square$.

8.1 is about $\square$. .3 is about $\square$.

5.4 is about $\square$. 2.2 is about $\square$.

The Nearest Tenth



.05 is halfway between .00 and .10, or halfway between .0 and .1.

.15 is halfway between .$\square\square$ and .20, or halfway between .$\square$ and .2.

.25 is halfway between .20 and .$\square\square$, or halfway between .2 and .$\square$.

.13 is nearer to .10 than to .20, so .13 is about .10 or .1.

.17 is nearer to .20 than to .$\square\square$, so .17 is about .20 or .2.

Round off to the nearest tenth.

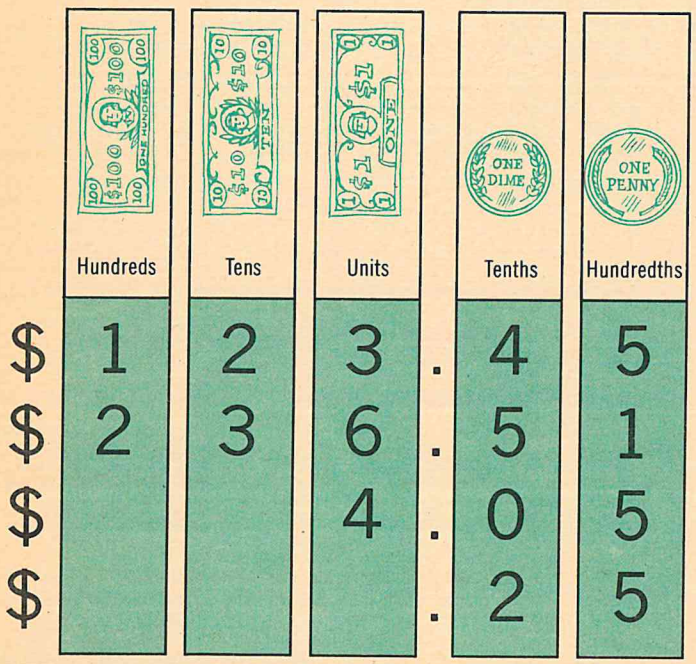
.08 is about .$\square$. .34 is about .$\square$.

.51 is about .$\square$. .67 is about .$\square$.

Place Value and United States Money

The money system of the United States matches the Arabic system of writing numerals.

Numerals	Sums of Money
1. = 1 unit.	\$1 = 1 dollar.
10. = 10 units = 1 ten.	\$10 = 10 dollars = 1 ten-dollar bill.
100. = 10 tens = 100 units = 1 hundred.	\$100 = 10 ten-dollar bills = 100 dollars = 1 hundred-dollar bill.
.1 = $\frac{1}{10}$ of a unit = 1 tenth.	\$.1 = $\frac{1}{10}$ of a dollar = 1 dime.
.01 = $\frac{1}{10}$ of a tenth = $\frac{1}{100}$ of a unit = 1 hundredth.	\$.01 = $\frac{1}{10}$ of a dime = $\frac{1}{100}$ of a dollar = 1 cent.



	Hundreds	Tens	Units	Tenths	Hundredths
\$	1	2	3 .	4	5
\$	2	3	6 .	5	1
\$			4 .	0	5
\$			.	2	5

\$123.45 means ☐ hundred-dollar bill + ☐ ten-dollar bills + ☐ one-dollar bills + ☐ dimes + ☐ cents.

\$236.51 means ☐ hundred-dollar bills + ☐ ten-dollar bills + ☐ one-dollar bills + ☐ dimes + ☐ cent.

\$4.05 means ☐ one-dollar bills + ☐ dimes + ☐ cents.

\$.25 means ☐ dimes + ☐ cents.

.1 = .10. 1 tenth = 10 hundredths.

\$.1 = \$.10. 1 dime = 10 cents.

.2 = .20. 2 tenths = ☐ hundredths.

\$.2 = \$.20. 2 dimes = ☐ cents.

.3 = . 3 tenths = ☐ hundredths.

\$.3 = \$. 3 dimes = ☐ cents.

.5 = . 5 tenths = ☐ hundredths.

\$.5 = \$. 5 dimes = ☐ cents.

.8 = . 8 tenths = ☐ hundredths.

\$.8 = \$. 8 dimes = ☐ cents.

Change to tenths.

$\frac{1}{2} = \frac{\square}{10} = .\square.$

$\frac{1}{5} = \frac{\square}{10} = .\square.$

$\frac{2}{5} = \frac{\square}{10} = .\square.$

$\frac{3}{5} = \frac{\square}{10} = .\square.$

$\frac{4}{5} = \frac{\square}{10} = .\square.$

Change to hundredths.

$\frac{1}{2} = \frac{\square}{100} = .\square\square.$

$\frac{1}{5} = \frac{\square}{100} = .\square\square.$

$\frac{2}{5} = \frac{\square}{100} = .\square\square.$

$\frac{3}{5} = \frac{\square}{100} = .\square\square.$

$\frac{4}{5} = \frac{\square}{100} = .\square\square.$

Change to hundredths.

$\frac{1}{4} = \frac{\square}{100} = .\square\square.$

$\frac{2}{4} = \frac{\square}{100} = .\square\square.$

$\frac{3}{4} = \frac{\square}{100} = .\square\square.$

$\frac{1}{10} = \frac{\square}{100} = .\square\square.$

$\frac{3}{10} = \frac{\square}{100} = .\square\square.$

$\frac{8}{10} = \frac{\square}{100} = .\square\square.$

Change to common fractions in lowest terms.

$.3 = \frac{\square}{10}.$

$.7 = \frac{\square}{\square}.$

$.9 = \frac{\square}{\square}.$

$.13 = \frac{\square}{100}.$

$.27 = \frac{\square}{\square}.$

$.5 = \frac{\square}{10} = \frac{\square}{\square}.$

$.8 = \frac{\square}{\square} = \frac{\square}{\square}.$

$.6 = \frac{\square}{\square} = \frac{\square}{\square}.$

$.25 = \frac{\square}{100} = \frac{\square}{\square}.$

$.50 = \frac{\square}{\square} = \frac{\square}{\square}.$

Adding Denominate Numbers with No Exchange

One squash weighed 1 lb. 2 oz. Another squash weighed 1 lb. 3 oz. How much did they weigh together?

To answer the question, add the pounds separately and add the ounces separately.

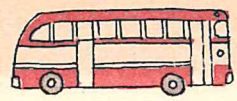
$$\begin{array}{r} 1 \text{ lb. } 2 \text{ oz.} \\ +1 \text{ lb. } 3 \text{ oz.} \\ \hline \end{array}$$

Answer: 2 lb. 5 oz.

Add.

2 lb. 4 oz. +8 lb. 3 oz. □ lb. □ oz.	8 ft. 4 in. +5 ft. 2 in. □ ft. □ in.	5 yd. 1 ft. +8 yd. 1 ft. □ yd. □ ft.
5 hr. 10 min. +2 hr. 15 min.	8 wk. 1 da. +7 wk. 2 da.	4 yr. 3 mo. +5 yr. 2 mo.
8 min. 10 sec. +6 min. 20 sec.	3 ft. 5 in. +10 ft. 4 in.	8 da. 5 hr. +4 da. 6 hr.

Adding Denominate Numbers with Exchange



A bus traveling between two cities makes one stop on the way. The first part of the trip takes 2 hr. 40 min. The second part takes 1 hr. 30 min. How long does the whole trip take?

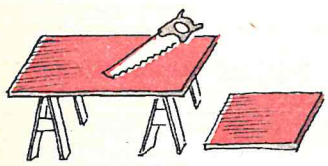
$$\begin{array}{r} 2 \text{ hr. } 40 \text{ min.} \\ +1 \text{ hr. } 30 \text{ min.} \\ \hline \end{array}$$

Exchange 60 min. for 1 hr.

3 hr. 70 min. = 3 hr. + 1 hr. 10 min. = 4 hr. 10 min.

5 hr. 50 min. +4 hr. 40 min. □ hr. □ min. = □ hr. + □ hr. □ min. = □ hr. □ min.	4 hr. 20 min. +3 hr. 40 min. □ hr. □ min. = □ hr. + □ hr. = □ hr.
6 ft. 8 in. +3 ft. 9 in. □ ft. □ in. = □ ft. + □ ft. □ in. = □ ft. □ in.	5 wk. 6 da. +3 wk. 4 da. □ wk. □ da. = □ wk. + □ wk. □ da. = □ wk. □ da.

Subtracting Denominate Numbers with No Exchange



A length of 2 ft. 1 in. was cut from a board that was 8 ft. 6 in. long. How long a piece was left? Subtract feet and inches separately.

$$\begin{array}{r} 8 \text{ ft. } 6 \text{ in.} \\ -2 \text{ ft. } 1 \text{ in.} \\ \hline \end{array}$$

Answer: 6 ft. 5 in.

Subtract.

10 ft. 7 in. −4 ft. 2 in. □ ft. □ in.	8 hr. 40 min. −2 hr. 10 min. □ hr. □ min.	6 wk. 5 da. −2 wk. 3 da. □ wk. □ da.
30 min. 45 sec. −10 min. 12 sec.	6 da. 10 hr. −2 da. 4 hr.	9 yd. 2 ft. −4 yd. 1 ft.

Subtracting Denominate Numbers with Exchange

5 ft. 4 in. = 4 ft. 16 in.
−1 ft. 8 in. = −1 ft. 8 in.
□ ft. □ in.

Exchange 1 ft. for 12 in.

9 hr. 15 min. = □ hr. □ min.
−3 hr. 40 min. = −3 hr. 40 min.
□ hr. □ min.

Exchange 1 hr. for □ min.

18 yd. 1 ft. = □ yd. □ ft.
−4 yd. 2 ft. = −4 yd. 2 ft.
□ yd. □ ft.

Exchange 1 yd. for □ ft.

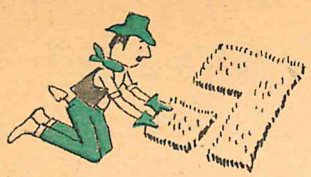
8 lb. 4 oz. = □ lb. □ oz.
−3 lb. 10 oz. = −3 lb. 10 oz.
□ lb. □ oz.

Exchange 1 lb. for □ oz.

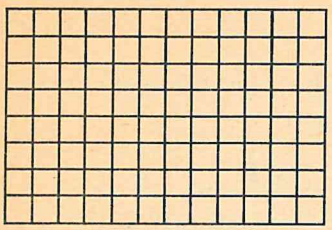
6 da. 4 hr. = □ da. □ hr.
−1 da. 12 hr. = −1 da. 12 hr.
□ da. □ hr.

Exchange 1 da. for □ hr.

Area of a Rectangle

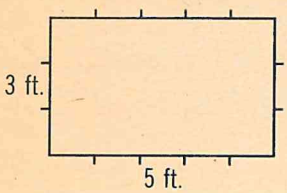


The gardener is building a lawn by putting down squares of sod. Each square is 1 ft. long and 1 ft. wide; this is called a **square foot.** He is covering a rectangle on the ground. The rectangle is 12 ft. long and 8 ft. wide. How many square feet will fill up the space in the rectangle? This number is called the **area** of the rectangle.



The rectangle is covered by 8 rows of squares with 12 squares in each row. The number of squares is $8 \times 12 = 96$. The area is 96 sq. ft.

To find the area of a rectangle in square units, multiply the number of units in the length by the number of units in the width.

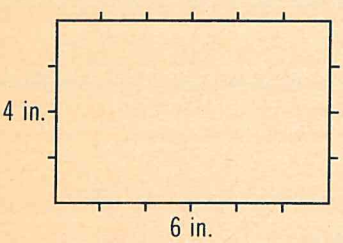


Find the number of square feet in a rectangle 5 ft. long and 3 ft. wide.

Area = ☐ ft. × ☐ ft.

= ☐ sq. ft.

To check your answer, divide the rectangle above into squares, and count the squares.



Find the number of square inches in a rectangle 6 in. long and 4 in. wide.

Area = ☐ in. × ☐ in.

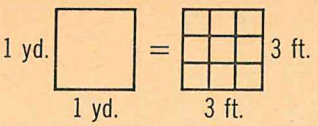
= ☐ sq. in.

To check your answer, divide the rectangle above into squares, and count the squares.

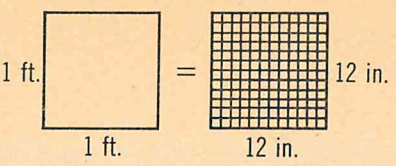
Find the number of square yards in a rectangle 15 yd. long and 8 yd. wide.

Find the number of square miles in a rectangle 10 mi. long and 3 mi. wide.

Area (Change of Unit)



1 sq. yd. = 1 yd. × 1 yd.
= 3 ft. × 3 ft.
= 9 sq. ft.



1 sq. ft.
= 1 ft. × 1 ft.
= 12 in. × 12 in.
= 144 sq. in.

A rectangle has dimensions 6 yd. by 2 yd. Find the number of square feet in its area.

First Way

6 yd. = ☐ ft.

2 yd. = ☐ ft.

Area = ☐ ft. × ☐ ft.

= ☐ sq. ft.

Second Way

Area = 6 yd. × 2 yd.

= ☐ sq. yd.

= ☐ × 9 sq. ft.

= ☐ sq. ft.

A rectangle has dimensions 3 ft. by 2 ft. Find the number of square inches in its area.

First Way

3 ft. = ☐ in.

2 ft. = ☐ in.

Area = ☐ in. × ☐ in.

= ☐ sq. in.

Second Way

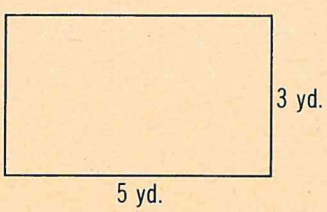
Area = 3 ft. × 2 ft.

= ☐ sq. ft.

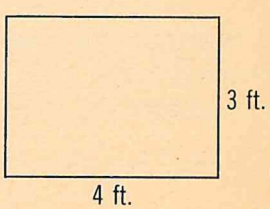
= ☐ × 144 sq. in.

= ☐ sq. in.

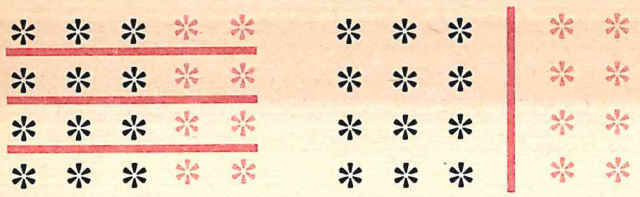
Find the number of square feet in a rectangle that is 5 yd. by 3 yd.



Find the number of square inches in a rectangle that is 4 ft. by 3 ft.



The Distributive Law



$4 \times (3 + 2) \quad = \quad 4 \times 3 \quad + \quad 4 \times 2$

The diagram above shows two ways of multiplying the sum of two numbers by a third number.

In the first way, the two numbers are added, $(3 + 2 = 5)$, and then their sum is multiplied by the third number, $(4 \times 5 = 20)$.

In the second way, each of the two numbers is multiplied by the third number $(4 \times 3 = 12$, and $4 \times 2 = 8)$, and then the products are added $(12 + 8 = 20)$. The fact that both ways give the same answer is called **the distributive law.**

Here are some more examples of the distributive law. Write the missing numbers where the statement is not complete.

$5 \times (6 + 8) = (5 \times 6) + (5 \times 8).$

$8 \times (2 + 3) = (8 \times 2) + (8 \times \square).$

$9 \times (6 + 10) = (9 \times \square) + (9 \times 10).$

$7 \times (2 + \square) = (7 \times 2) + (7 \times 9).$

$4 \times (\square + 6) = (4 \times 5) + (4 \times 6).$

$10 \times (2 + 12) = (10 \times \square) + (10 \times \square).$

$11 \times (\square + \square) = (11 \times 6) + (11 \times 4).$

On page 3 you used the distributive law to think out the product of 4×22. First you wrote 22 as the sum of two numbers, $20 + 2$:

$$4 \times 22 = 4 \times (20 + 2) = (4 \times 20) + (4 \times 2)$$
$$= 80 + 8 = 88.$$

Use the distributive law to think out these products.

$3 \times 13 = 3 \times (\square + \square) = (3 \times \square) + (3 \times \square)$

$= \square + \square = \square.$

$5 \times 12 = 5 \times (\square + \square) = (5 \times \square) + (5 \times \square)$

$= \square + \square = \square.$

Write the missing numbers in these examples of the commutative law of addition:

$15 + 4 = 4 + \square.$ $\qquad$ $8 + 9 = \square + 8.$

$12 + 5 = \square + \square.$ $\qquad$ $9 + 40 = \square + \square.$

Write the missing numbers in these examples of the commutative law of multiplication:

$21 \times 3 = 3 \times \square.$ $\qquad$ $100 \times 5 = \square \times 100.$

$20 \times 4 = \square \times \square.$ $\qquad$ $36 \times 2 = \square \times \square.$

Put in the missing parentheses in these examples of the associative law of addition:

$(5 + 6) + 7 = 5 + 6 + 7.$

$3 + 10 + 2 = 3 + (10 + 2).$

$4 + 5 + 9 = 4 + 5 + 9.$

Put in the missing parentheses in these examples of the associative law of multiplication.

$(8 \times 2) \times 3 = 8 \times 2 \times 3.$

$5 \times 4 \times 10 = 5 \times (4 \times 10).$

$10 \times 2 \times 1 = 10 \times 2 \times 1.$

In $(4 + 7) + 9$, the parentheses tell you to add $\square$ and $\square$ first.

In $4 + (7 + 9)$, the parentheses tell you to add $\square$ and $\square$ first.

In $5 \times (10 \times 2)$, the parentheses tell you to multiply $\square$ and $\square$ first.

In $(5 \times 10) \times 2$, the parentheses tell you to multiply $\square$ and $\square$ first.

Underline the right words.

To calculate $5 \times (11 + 3)$, first you (add multiply), then you (add multiply).

To calculate $(5 \times 11) + (5 \times 3)$, first you (add multiply), then you (add multiply).

You have learned five laws obeyed by numbers. Use abbreviations for these laws as follows: CA, CM, AA, AM, and D. (CA means commutative law of addition, etc.) Next to each example below, name the law that fits.

$5 \times (10 + 6) = (5 \times 10) + (5 \times 6)$ $\square$ $\qquad$ $5 \times 7 = 7 \times 5$ $\square$

$(4 \times 2) \times 3 = 4 \times (2 \times 3)$ $\square$ $\qquad$ $10 + 2 = 2 + 10$ $\square$

$(6 + 2) + 9 = 6 + (2 + 9)$ $\square$ $\qquad$ $11 \times 1 = 1 \times 11$ $\square$

Three-Digit Multiplier (Higher Hundreds)

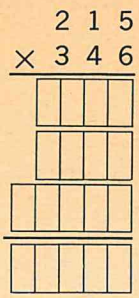
Arrange your work in the short way.

Step I—Multiply by 6.

Step II—Multiply by 40.

Step III—Multiply by 300.

Step IV—Add.



Multiply, arranging your work in the short way.

420 ×516	815 ×207
634 ×415	108 ×327
540 ×420	369 ×305

The sum 325 + 100 can be thought out in these steps:

I. 325 + 100 = 100 + 325. ☐

II. 100 + 325 = 100 + (300 + 25).

III. 100 + (300 + 25) = (100 + 300) + 25. ☐

IV. (100 + 300) + 25 = 400 + 25.

V. 400 + 25 = 425.

Which law is used in Step I? in Step III? (CA, CM, AA, AM, or D). Write the answers in the boxes above.

The product 20 × 3 can be thought out in these steps:

I. 20 × 3 = (2 × 10) × 3.

II. (2 × 10) × 3 = 2 × (10 × 3). ☐

III. 2 × (10 × 3) = 2 × 30.

IV. 2 × 30 = 60.

Which law is used in step II? (CA, CM, AA, AM, or D). Write the answer in the box above.

The product 12 × 15 can be thought out in these steps:

I. 12 × 15 = 12 × (10 + 5).

II. 12 × (10 + 5) = (12 × 10) + (12 × 5). ☐

III. (12 × 10) + (12 × 5) = (10 × 12) + (5 × 12). ☐

IV. (10 × 12) + (5 × 12) = 120 + (5 × 12).

V. 120 + (5 × 12) = 120 + [5 × (10 + 2)].

VI. 120+[5×(10+2)]=120+[(5×10)+(5×2)]. ☐

VII. 120+[(5×10)+(5×2)]=120+(50+10).

VIII. 120 + (50 + 10) = 120 + 60 = 180.

Which law is used in Step II? in Step III? in Step VI? Write the answers in the boxes above.

The product 50 × 30 can be thought out in these steps:

I. 50 × 30 = 50 × (3 × 10).

II. 50 × (3 × 10) = (50 × 3) × 10. ☐

III. (50 × 3) × 10 = (3 × 50) × 10. ☐

IV. (3 × 50) × 10 = [3 × (5 × 10)] × 10.

V. [3 × (5 × 10)] × 10 = [(3 × 5) × 10] × 10. ☐

VI. [(3 × 5) × 10] × 10 = (3 × 5) × (10 × 10). ☐

VII. (3 × 5) × (10 × 10) = 15 × 100 = 1500.

Which law is used in Step II? in III? in V? in VI? Write the answers in the boxes above.

Using the numbers 2 and 5, write an example of the commutative law of addition.

Using the numbers 6 and 9, write an example of the commutative law of multiplication.

Using the numbers 2, 3, and 5, write an example of the distributive law.

A Two-Digit Divisor and a Four- and Five-Digit Dividend

32 ×100 = ☐

32 ×200 = ☐

32 ×300 = ☐

32 ×10 = ☐

32 ×20 = ☐

32 ×3 = ☐

32 ×4 = ☐

32 ×5 = ☐

32)6 8 8 0

Answer: ☐

32 ×100 = ☐

(too big)

32 ×30 = ☐

32 ×40 = ☐

32 ×50 = ☐

32 ×5 = ☐

32)1 4 4 0

Answer: ☐

1000 × 16 = ☐

2000 × 16 = ☐

3000 × 16 = ☐

16 ×100 = ☐

16 ×200 = ☐

16 ×30 = ☐

16 ×40 = ☐

16 ×50 = ☐

16 ×6 = ☐

16 ×7 = ☐

16 ×8 = ☐

16)3 4 3 6 8

Answer: ☐

Multiplying by a Multiple of Ten Thousand

$10 \times 10 = 100$. $10 \times 10 \times 10 = 10 \times 100 = 1000$.

$10 \times 10 \times 10 \times 10 = 10 \times 1000 = 10{,}000$.

$10{,}000 \times 2 = (10 \times 10 \times 10 \times 10) \times 2$.

At the right you see what happens when you

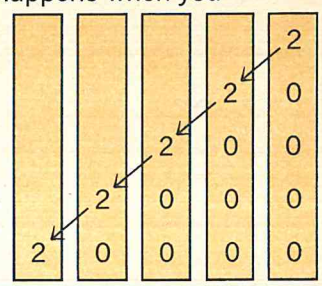
multiply 2 by 10:

multiply 20 by 10:

multiply 200 by 10:

multiply 2000 by 10:

So $10{,}000 \times 2 = 20{,}000$.



To multiply a number by 10,000 move each digit 4 places to the left. Zeros hold the units place, the tens place, the hundreds place, and the thousands place.

$10{,}000 \times 5 =$ ☐. $10{,}000 \times 7 =$ ☐.

$10{,}000 \times 12 =$ ☐. $10{,}000 \times 26 =$ ☐.

$30{,}000 \times 4 = (3 \times 10{,}000) \times 4 = 3 \times (10{,}000 \times 4)$
$= 3 \times 40{,}000 = 120{,}000$.

Multiplying by 30,000 is like multiplying first by 10,000 and then by 3.

4
×30,000
1 2 0,0 0 0

First put down 4 zeros and move 4 places left.

Then multiply by 3.

Six-Digit Dividend

$10{,}000 \times 18 =$ ☐.

$20{,}000 \times 18 =$ ☐.

18)3 9 8 0 1 6

Answer: ☐

Common Denominators in Unrelated Fractions

Change $\frac{1}{3}$ and $\frac{1}{4}$ to fractions with a common denominator.

$\frac{1}{3}$ may be changed to fractions with denominators 3, 6, 9, 12, 15, . . . (Count by threes.)

$\frac{1}{4}$ may be changed to fractions with denominators 4, 8, 12, 16, 20, . . . (Count by fours.)

The lowest number that is in both lists is 12. 12 is the lowest common denominator.

$\frac{1}{3} = \frac{\square}{12}$. $\frac{1}{4} = \frac{\square}{12}$.

Change each pair of fractions to fractions with a common denominator.

$\frac{1}{6}$ and $\frac{1}{4}$

Count by sixes. 6, $\square$, $\square$, $\square$, $\square$, . . .

Count by fours. 4, $\square$, $\square$, $\square$, $\square$, . . .

The lowest common denominator is $\square$.

$\frac{1}{6} = \frac{\square}{\square}$. $\frac{1}{4} = \frac{\square}{\square}$.

$\frac{3}{4}$ and $\frac{7}{10}$

Count by fours. 4, $\square$, $\square$, $\square$, $\square$, . . .

Count by tens. 10, $\square$, $\square$, $\square$, $\square$, . . .

The lowest common denominator is $\square$.

$\frac{3}{4} = \frac{\square}{\square}$. $\frac{7}{10} = \frac{\square}{\square}$.

$\frac{5}{6}$ and $\frac{2}{9}$

Count by sixes. 6, $\square$, $\square$, $\square$, $\square$, . . .

Count by nines. 9, $\square$, $\square$, $\square$, $\square$, . . .

The lowest common denominator is $\square$.

$\frac{5}{6} = \frac{\square}{\square}$. $\frac{2}{9} = \frac{\square}{\square}$.

$\frac{2}{7}$ and $\frac{3}{5}$

Count by sevens. 7, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, . . .

Count by fives. 5, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, . . .

The lowest common denominator is $\square$.

$\frac{2}{7} = \frac{\square}{\square}$. $\frac{3}{5} = \frac{\square}{\square}$.

Change $\frac{1}{6}$, $\frac{1}{4}$, and $\frac{1}{3}$ to fractions with a common denominator.

Step I Pick out the largest denominator from among 6, 4, and 3. It is 6.

Step II Count by sixes: 6, 12, 18, 24, 30, 36, 42, 48, . . .

Step III Stop at the first number that is a common denominator for 4 and 3. It is 12.

$\frac{1}{6} = \frac{\square}{12}$. $\frac{1}{4} = \frac{\square}{12}$. $\frac{1}{3} = \frac{\square}{12}$.

Change each set of fractions to fractions with a common denominator.

$\frac{1}{4}, \frac{1}{3}, \frac{1}{2}$ 4 is the highest denominator.

Count by 4's. 4, $\square$, $\square$, $\square$, $\square$, $\square$, . . .

The lowest common denominator is $\square$.

$\frac{1}{4} = \frac{\square}{\square}$. $\frac{1}{3} = \frac{\square}{\square}$. $\frac{1}{2} = \frac{\square}{\square}$.

$\frac{1}{2}, \frac{2}{3}, \frac{1}{5}$ Count by 5's.

5, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, . . .

The lowest common denominator is $\square$.

$\frac{1}{2} = \frac{\square}{\square}$. $\frac{2}{3} = \frac{\square}{\square}$. $\frac{1}{5} = \frac{\square}{\square}$.

$\frac{2}{5}, \frac{3}{4}, \frac{1}{2}$ Count by $\square$'s.

$\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, . . .

The lowest common denominator is $\square$.

$\frac{2}{5} = \frac{\square}{\square}$. $\frac{3}{4} = \frac{\square}{\square}$. $\frac{1}{2} = \frac{\square}{\square}$.

$\frac{2}{3}, \frac{5}{8}, \frac{1}{6}$ Count by $\square$'s.

$\square$, $\square$, $\square$, $\square$, $\square$, $\square$, . . .

The lowest common denominator is $\square$.

$\frac{2}{3} = \frac{\square}{\square}$. $\frac{5}{8} = \frac{\square}{\square}$. $\frac{1}{6} = \frac{\square}{\square}$.

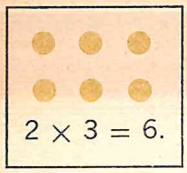
$\frac{1}{3}, \frac{3}{5}, \frac{3}{4}$ Count by $\square$'s.

$\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, . . .

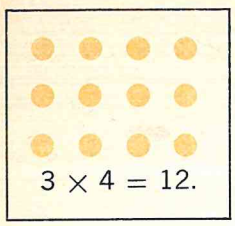
The lowest common denominator is $\square$.

$\frac{1}{3} = \frac{\square}{\square}$. $\frac{3}{5} = \frac{\square}{\square}$. $\frac{3}{4} = \frac{\square}{\square}$.

Factors of a Whole Number



6 checkers can be arranged in 2 equal rows with 3 checkers in each row. This shows that $2 \times 3 = 6$. The numbers 2 and 3 are called **factors** of 6.

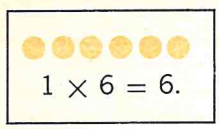


12 checkers can be arranged in 3 equal rows with 4 checkers in each row. This shows that $3 \times 4 = 12$. 3 and 4 are called **factors** of 12.

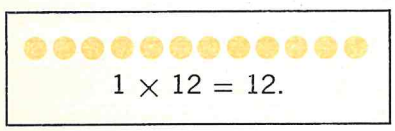
If a number of checkers can be arranged in equal rows, the number of rows and the number of checkers in each row is called a factor of the number of checkers.

We can also explain what the word factor means in this way:

When two numbers are multiplied, each of the numbers is called a factor of the product.



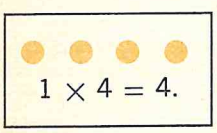
6 checkers can also be arranged in 1 row, because $1 \times 6 = 6$. So 1 is a factor of 6, and 6 is a factor of 6.

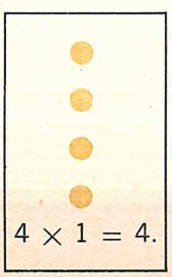


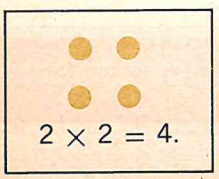
12 checkers can be arranged in 1 row. So 1 is a factor of 12, and 12 is a factor of 12.

Any number of checkers can be arranged in 1 row, with all the checkers in the row. So **1 is a factor of any whole number,** and **any whole number is a factor of itself.**

Find all the factors of 4. To answer the question, arrange 4 checkers in 1 row, and also in several equal rows.



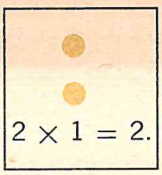


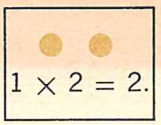


These are the only possible arrangements. The factors of 4 are 1, 2, and 4.

Prime Numbers

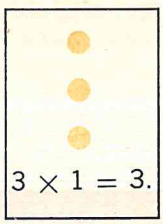
Find all the factors of 2.

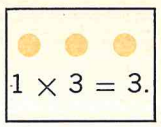




These are the only possible arrangements. The factors of 2 are 1 and 2.

Find all the factors of 3.





These are the only possible arrangements. The factors of 3 are 1 and 3.

Arrange 5 dots in 1 row or in equal rows in all possible ways. Write the product pictured by each arrangement. Then list all the factors of 5.

The only factors of 2 are 1 and 2.

The only factors of 3 are ☐ and ☐.

The only factors of 5 are ☐ and ☐.

The only factors of each of these numbers are 1 and the number itself. Numbers higher than 1 for which this is true are called **prime numbers.** 2, 3, and 5 are prime numbers. What is the next higher prime number after 5?

Answer: ☐

If a number is not prime, it has a factor different from 1 and itself. Complete each of the statements below by writing in the box a factor different from 1 or the number itself.

4 is not prime because ☐ is a factor of 4.

6 is not prime because ☐ is a factor of 6.

8 is not prime because ☐ is a factor of 8.

9 is not prime because ☐ is a factor of 9.

10 is not prime because ☐ is a factor of 10.

Finding Prime Numbers

Here is a quick way of finding all the prime numbers that are less than 25. The numbers from 2 to 25 are listed at the left. Put a circle around 2 to show that it is prime. Then count by 2's and cross off each number you get. The numbers like 4, 6, 8, and so on, are not prime, because 2 is a factor of each of them.

2	3	4	5	6	7
8	9	10	11	12	13
14	15	16	17	18	19
20	21	22	23	24	25

Now find the first number after 2 that is not crossed off. It is 3. Put a circle around 3 to show that it is prime. Then count by 3's and cross off each number you get. You cross off 6, 9, 12, and so on.

Now find the first number after 3 that is not crossed off. It is 5. Circle the 5. Then count by 5's, and cross off all the multiples of 5.

Continue in this way. Each time you find a new prime number, cross off its multiples. The first number after it that is not crossed off is the next prime number.

After you have found all the prime numbers that are less than 25, list them here: 2, 3, 5, 7, ☐, ☐, ☐, ☐, ☐, ☐.

Memorize this list.

☐ and ☐ left
$2\overline{)11}$

☐ and ☐ left
$3\overline{)11}$

☐ and ☐ left
$5\overline{)11}$

☐ and ☐ left
$7\overline{)11}$

In these exercises, 11 is divided by every prime number less than 11. Each remainder is not 0. This proves that 11 is prime.

Prime Factors

$12 = 4 \times 3$.

The factor 3 is prime. The factor 4 is not prime.

But $4 = 2 \times 2$.

Replacing 4 by 2×2, we get: $12 = 2 \times 2 \times 3$.

Now 12 is written as a product of factors all of which are prime.

This can be done for every number that is not prime. Here is an easy way of doing it:

Example To write 60 as a product of prime factors: The smallest prime number 2 is a factor of 60. Divide 60 by 2. The quotient is 30. 30 is not prime.

Start here. → $2\overline{)60}$

Now divide 30 by 2. The quotient is 15. 15 is not prime. But 2 is not a factor of 15.

$2\overline{)30}$

Try the next prime number 3. 3 is a factor of 15. Divide 15 by 3. The quotient is 5. 5 is prime, so stop here.

$3\overline{)15}$

5

The divisors you used and the last quotient are the prime factors of 60:

$60 = 2 \times 2 \times 3 \times 5$ (2 is written twice because you used it as a divisor twice.)

Write 24 as a product of primes.

☐
☐)☐
☐)☐
$2\overline{)24}$

$24 =$ ☐ × ☐ × ☐ × ☐.

Write 12 as a product of primes.

☐
☐)☐
☐)12

$12 =$ ☐ × ☐ × ☐.

Write 18 as a product of primes.

☐
☐)☐
☐)18

$18 =$ ☐ × ☐ × ☐.

Using Prime Factors to Find a Common Denominator

Change $\frac{1}{8}$ and $\frac{3}{10}$ to fractions with a common denominator. First write 8 and 10 as products of prime factors.

$8 = 2 \times 2 \times 2.$ $\quad 10 = 2 \times 5.$

Form a new product using each prime factor the highest number of times it is used in one of the old products. Use 2 three times and 5 once: $2 \times 2 \times 2 \times 5 = 40$, which is the least common denominator.

$\frac{1}{8} = \frac{\square}{40}.$ $\quad \frac{3}{10} = \frac{\square}{40}.$

Change to fractions with a common denominator.

$\frac{1}{6}$ and $\frac{3}{4}$

$6 = 2 \times 3.$

$4 = 2 \times 2.$

$2 \times 2 \times 3 = \square.$

$\frac{1}{6} = \frac{\square}{\square}.$ $\quad \frac{3}{4} = \frac{\square}{\square}.$

$\frac{1}{10}$ and $\frac{2}{15}$

$10 = 2 \times 5.$

$15 = 3 \times 5.$

$2 \times 3 \times 5 = \square.$

$\frac{1}{10} = \frac{\square}{\square}.$ $\quad \frac{2}{15} = \frac{\square}{\square}.$

$\frac{1}{14}$ and $\frac{5}{21}$

$14 = \square \times \square.$

$21 = \square \times \square.$

$\square \times \square \times \square = \square.$

$\frac{1}{14} = \frac{\square}{\square}.$ $\quad \frac{5}{21} = \frac{\square}{\square}.$

$\frac{1}{8}$, $\frac{5}{12}$, and $\frac{7}{9}$

$8 = \square \times \square \times \square.$

$12 = \square \times \square \times \square.$

$9 = \square \times \square.$

$\square \times \square \times \square \times \square \times \square = \square.$

$\frac{1}{8} = \frac{\square}{\square}.$ $\quad \frac{5}{12} = \frac{\square}{\square}.$ $\quad \frac{7}{9} = \frac{\square}{\square}.$

$\frac{5}{6}$, $\frac{3}{8}$, and $\frac{1}{2}$

$6 = \square \times \square.$

$8 = \square \times \square \times \square.$

$2 = 2.$

$\square \times \square \times \square \times \square = \square.$

$\frac{5}{6} = \frac{\square}{\square}.$ $\quad \frac{3}{8} = \frac{\square}{\square}.$ $\quad \frac{1}{2} = \frac{\square}{\square}.$

$\frac{1}{3}$, $\frac{1}{2}$, and $\frac{3}{10}$

$3 = \square.$

$2 = \square.$

$10 = \square \times \square.$

$\square \times \square \times \square = \square.$

$\frac{1}{3} = \frac{\square}{\square}.$ $\quad \frac{1}{2} = \frac{\square}{\square}.$ $\quad \frac{3}{10} = \frac{\square}{\square}.$

$\frac{2}{9}$, $\frac{1}{6}$, and $\frac{3}{4}$

$9 = \square \times \square.$

$6 = \square \times \square.$

$4 = \square \times \square.$

$\square \times \square \times \square \times \square = \square.$

$\frac{2}{9} = \frac{\square}{\square}.$ $\quad \frac{1}{6} = \frac{\square}{\square}.$ $\quad \frac{3}{4} = \frac{\square}{\square}.$

$\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{12}$

$4 = \square \times \square.$

$8 = \square \times \square \times \square.$

$12 = \square \times \square \times \square.$

$\square \times \square \times \square \times \square = \square.$

$\frac{1}{4} = \frac{\square}{\square}.$ $\quad \frac{1}{8} = \frac{\square}{\square}.$ $\quad \frac{1}{12} = \frac{\square}{\square}.$

$\frac{1}{2}$ and $\frac{3}{5}$

$2 =$

$5 =$

$= \square.$

$\frac{1}{2} = ____.$ $\quad \frac{3}{5} = ____.$

$\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{6}$

$3 =$

$4 =$

$6 =$

$= \square.$

$\frac{2}{3} = \frac{\square}{\square}.$ $\quad \frac{3}{4} = \frac{\square}{\square}.$ $\quad \frac{1}{6} = \frac{\square}{\square}.$

Addition of Fractions

$6 = 2 \times 3.$
$4 = 2 \times 2.$
$2 \times 2 \times 3 = 12.$

$$5\tfrac{5}{6} = 5\tfrac{10}{12}$$
$$+4\tfrac{3}{4} = 4\tfrac{9}{12}$$
$$9\tfrac{19}{12} = 10\tfrac{7}{12}.$$

$5 = \square.$
$3 = \square.$
$\square \times \square = \square.$

$$8\tfrac{1}{5} = 8\tfrac{\square}{\square}$$
$$+12\tfrac{2}{3} = 12\tfrac{\square}{\square}$$
$$\frac{\square}{\square}$$

$9 = \square \times \square.$
$12 = \square \times \square \times \square.$
$\square \times \square \times \square \times \square = \square.$

$$7\tfrac{1}{9} = 7\tfrac{\square}{\square}$$
$$+3\tfrac{5}{12} = 3\tfrac{\square}{\square}$$
$$\frac{\square}{\square}$$

$2 = \square.$
$3 = \square.$
$6 = \square \times \square.$
$\square \times \square = \square.$

$$8\tfrac{1}{2} = 8\tfrac{\square}{\square}$$
$$3\tfrac{1}{3} = 3\tfrac{\square}{\square}$$
$$+5\tfrac{1}{6} = 5\tfrac{\square}{\square}$$
$$\square\tfrac{\square}{\square} = \square.$$

$$8\tfrac{1}{12} + 2\tfrac{3}{4} + 5\tfrac{1}{3}$$

$$3\tfrac{1}{5} + 9\tfrac{1}{2} + 6\tfrac{7}{10}$$

$$9\tfrac{1}{6} + 2\tfrac{1}{10}$$

$$7\tfrac{1}{3} + 10\tfrac{3}{4}$$

Subtraction of Fractions

$3 = 3.$
$4 = 2 \times 2.$
$2 \times 2 \times 3 = 12.$

$$8\tfrac{2}{3} = 8\tfrac{8}{12}$$
$$-5\tfrac{1}{4} = 5\tfrac{3}{12}$$
$$3\tfrac{5}{12}$$

$6 = \square \times \square.$
$10 = \square \times \square.$
$\square \times \square \times \square = \square.$

$$10\tfrac{5}{6} = 10\tfrac{\square}{\square}$$
$$-4\tfrac{3}{10} = 4\tfrac{\square}{\square}$$
$$\frac{\square}{\square} = \square\tfrac{\square}{\square}.$$

$4 = \square \times \square.$
$10 = \square \times \square.$
$\square \times \square \times \square = \square.$

$$8\tfrac{1}{4} = 8\tfrac{\square}{\square} = 7\tfrac{\square}{\square}$$
$$-1\tfrac{7}{10} = 1\tfrac{\square}{\square} = 1\tfrac{\square}{\square}$$
$$\square\tfrac{\square}{\square}$$

$5 = \square.$
$2 = \square.$
$\square \times \square = \square.$

$$12\tfrac{1}{5} = 12\tfrac{\square}{\square} = 11\tfrac{\square}{\square}$$
$$-3\tfrac{1}{2} = 3\tfrac{\square}{\square} = 3\tfrac{\square}{\square}$$
$$\square\tfrac{\square}{\square}$$

$$21\tfrac{1}{3} - 4\tfrac{1}{4}$$

$$7\tfrac{1}{4} - 2\tfrac{1}{5}$$

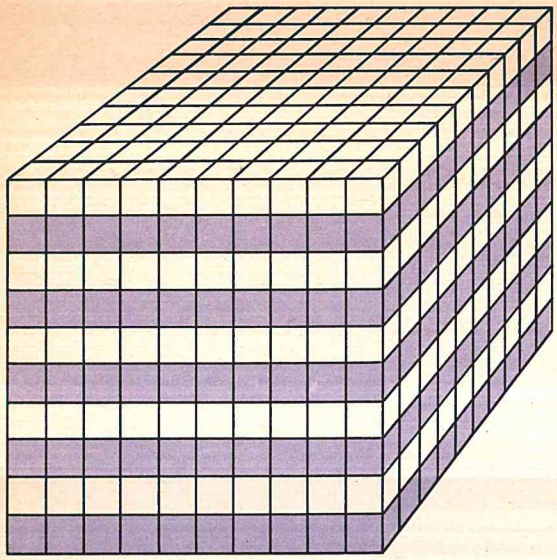
$$8\tfrac{1}{6} - 2\tfrac{1}{5}$$

$$19\tfrac{1}{3} - 4\tfrac{1}{2}$$

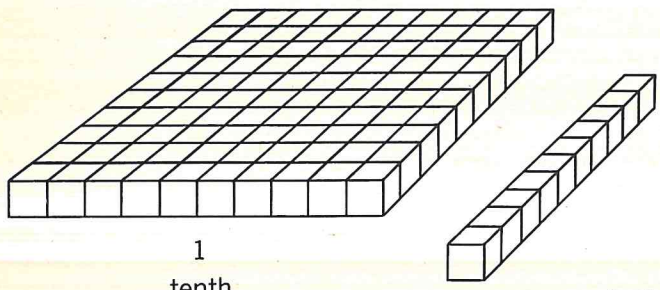
$\frac{7}{8} - \frac{1}{6} = \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}.$

$\frac{5}{8} - \frac{3}{10} = \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}.$

Decimal Fractions



1 unit



1 tenth

1 hundredth

1 thousandth

The large cube stands for 1 unit. The cube is divided into 10 layers. Each layer is 1 tenth of a unit. ☐ tenths = 1.

Each layer is divided into 10 strips. 10 layers with 10 strips in each layer contain 100 strips. Each strip is 1 hundredth of a unit. ☐ hundredths = 1.

☐ hundredths = 1 tenth.

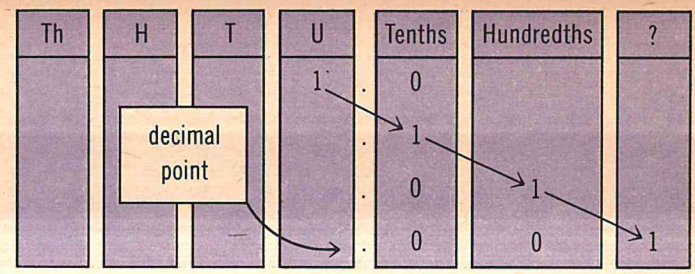
Each strip is divided into 10 small cubes. 100 strips with 10 cubes in each strip contain 100 × 10 or 1000 small cubes. Each small cube is 1 thousandth of a unit.

☐ thousandths = 1.

☐ thousandths = 1 tenth.

☐ thousandths = 1 hundredth.

1 hundredth is ☐ times as large as 1 thousandth. 1 thousandth is $\frac{1}{10}$ as large as 1 hundredth. 1 tenth is ☐ times as large as 1 thousandth. 1 thousandth is $\frac{1}{100}$ as large as 1 tenth.



When a digit moves one place to the right it becomes worth $\frac{1}{10}$ as much. A new place has been put to the right of the hundredths place. A 1 in this place is worth $\frac{1}{10}$ of 1 hundredth, or 1 thousandth. The new place is the thousandths place. When no other digit is in the tenths place or hundredths place, we put 0 in to hold the place.

.001 is another way of writing $\frac{1}{1000}$.

.002 is another way of writing $\frac{\square}{1000}$.

To read .005 say: ☐ thousandths.

To read .009 say: ☐ thousandths.

2 hundredths = 20 thousandths.

3 hundredths = ☐ thousandths.

5 hundredths = ☐ thousandths.

.024 = 2 hundredths + 4 thousandths

= 20 thousandths + 4 thousandths = 24 thousandths.

To read .024, say: 24 thousandths.

To read .036, say: ☐ thousandths.

.135 = 1 tenth + 3 hundredths + 5 thousandths

= 100 thousandths + 30 thousandths + 5 thousandths

= 135 thousandths.

To read .135, say: 135 thousandths.

To read .247, say: ☐ thousandths.

Write each number as a decimal.

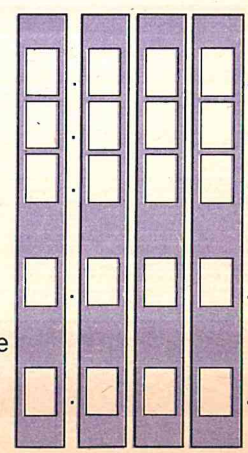
five and six thousandths =

three and fourteen thousandths =

six and ten thousandths =

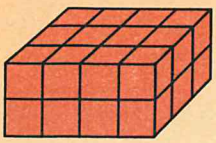
eight and one hundred sixty-four thousandths =

nine and three hundred twenty-one thousandths =



Volume of a Rectangular Solid

A cube that is 1 inch long, 1 inch wide, and 1 inch high is called a **cubic inch.**

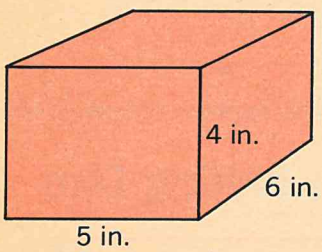


This rectangular solid is made of 2 layers of cubes, each of which is a cubic inch. Each layer has 3×4 cubic inches in it. The two layers have $2 \times 3 \times 4$ or 24 cubic inches.

The number of cubic units in a solid is called its **volume.**

To find the volume of a rectangular solid, multiply the number of units in the length, width, and height.

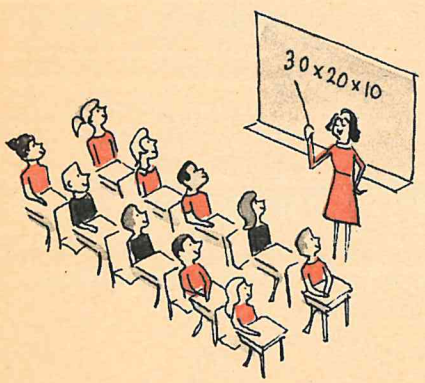
Find the number of cubic inches in a box that is 5 in. long, 6 in. wide, and 4 in. high.



Volume = ☐ in. × ☐ in. × ☐ in.

= ☐ cu. in.

A classroom is 30 ft. long, 20 ft. wide, and 10 ft. high. Find the number of cubic feet in the room.

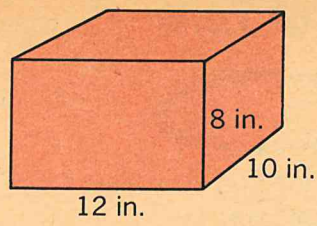


Volume = ☐ ft. × ☐ ft. × ☐ ft.

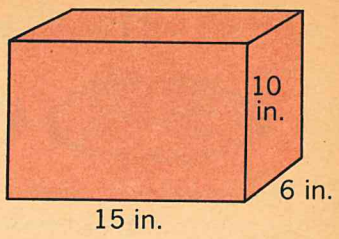
= ☐ cu. ft.

If there are 20 people in the room, about how many cubic feet of air are there for each person?

☐ ÷ ☐ = ☐



A



B

Box A is 12 in. long, 10 in. wide, and 8 in. high. Box B is 15 in. long, 6 in. wide, and 10 in. high. Which box has the larger volume? How much larger is the volume?

The volume of box A

= ☐ × ☐ × ☐ cu. in.

= ☐ cu. in.

The volume of box B

= ☐ × ☐ × ☐ cu. in.

= ☐ cu. in.

Box ☐ has the larger volume.

The volume is ☐ cu. in. larger.

1 cubic yard = 1 yd. × 1 yd. × 1 yd.

= 3 ft. × 3 ft. × 3 ft. = ☐ cu. ft.

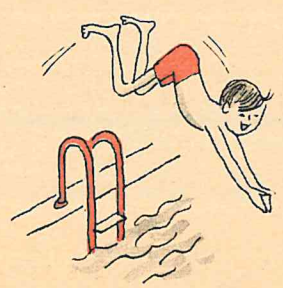
27	27	27	27
×2	×3	×4	×5
☐	☐	☐	☐

2 cu. yd. = ☐ cu. ft.

3 cu. yd. = ☐ cu. ft.

4 cu. yd. = ☐ cu. ft.

5 cu. yd. = ☐ cu. ft.



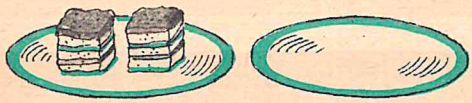
A swimming pool that is 50 ft. long and 25 ft. wide was filled to an average depth of 8 ft. How many cubic feet of water were in the pool?

Volume

= ☐ ft. × ☐ ft. × ☐ ft.

= ☐ cu. ft.

The Law of Zero



$2 + 0$

The number of cakes on both plates is $2 + 0 = 2$. This is an example of **the law of zero:**

When zero is added to a number, you get the same number as sum.

Add.

$15 + 0 =$ ☐.

$0 + 21 =$ ☐.

$0 + \frac{1}{2} =$ ☐.

$0 + 1\frac{3}{4} =$ ☐.

$0 + 2.1 =$ ☐.

$0 + .9 =$ ☐.

$16{,}000 + 0 =$ ☐.

$0 + 42{,}015 =$ ☐.

$\frac{2}{3} + 0 =$ ☐.

$4\frac{1}{5} + 0 =$ ☐.

$6.3 + 0 =$ ☐.

$.5 + 0 =$ ☐.

$5 + 3 - 3 = 5 + (3 - 3) = 5 + 0 =$ ☐.

$19 + 12 - 12 = 19 + (12 - 12) = 19 +$ ☐ $=$ ☐.

$45 + 27 - 27 = 45 + (27 - 27) = 45 +$ ☐ $=$ ☐.

$64 + 29 - 29 = 64 + (29 - 29) = 64 +$ ☐ $=$ ☐.

Zero as a Multiplier

○ ○ ○ ○ ○ ○	○ ○ ○ ○	○ ○	
$3 \times 2 =$ ☐.	$2 \times 2 =$ ☐.	$1 \times 2 =$ ☐.	$0 \times 2 =$ ☐.

When a number is multiplied by zero, the product is zero.

Multiply.

$0 \times 5 =$ ☐.

$0 \times 5{,}000 =$ ☐.

$0 \times \frac{1}{2} =$ ☐.

$4\frac{2}{7} \times 0 =$ ☐.

$0 \times 2.3 =$ ☐.

$6.25 \times 0 =$ ☐.

$0 \times 1 =$ ☐.

$0 \times 10 =$ ☐.

$25 \times 0 =$ ☐.

$6{,}243 \times 0 =$ ☐.

$\frac{3}{5} \times 0 =$ ☐.

$0 \times 3\frac{3}{4} =$ ☐.

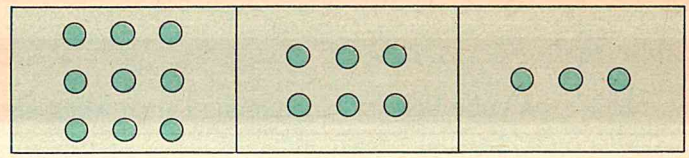
$5.7 \times 0 =$ ☐.

$.003 \times 0 =$ ☐.

$0 \times 0 =$ ☐.

$0 \times 1000 =$ ☐.

The Law of One



$3 \times 3 =$ ☐. $2 \times 3 =$ ☐. $1 \times 3 =$ ☐.

Here you see an example of **the law of one:**

When a number is multiplied by one, you get the same number as product.

Multiply.

$1 \times 31 =$ ☐.

$1 \times 5{,}000 =$ ☐.

$1 \times \frac{4}{5} =$ ☐.

$8\frac{2}{3} \times 1 =$ ☐.

$2.3 \times 1 =$ ☐.

$1 \times 4.56 =$ ☐.

$48 \times 1 =$ ☐.

$926 \times 1 =$ ☐.

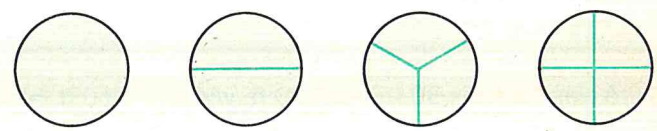
$\frac{6}{7} \times 1 =$ ☐.

$1 \times 4\frac{1}{4} =$ ☐.

$1 \times .02 =$ ☐.

$3.05 \times 1 =$ ☐.

Many Ways of Writing One



1 whole = ☐ halves = ☐ thirds = ☐ fourths.

$1 = \frac{\square}{2} = \frac{\square}{3} = \frac{\square}{4}$.

A fraction is equal to 1 if the numerator and denominator are equal.

Write the missing numbers.

$1 = \frac{\square}{8}$. $1 = \frac{\square}{12}$. $1 = \frac{\square}{15}$.

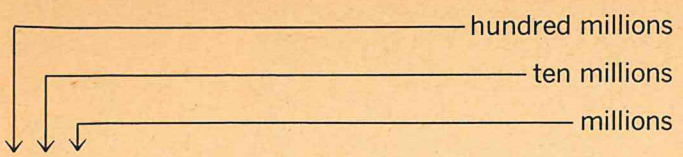
$1 = \frac{16}{\square}$. $1 = \frac{7}{\square}$. $1 = \frac{10}{\square}$.

$6 \times \frac{3}{3} = 6 \times 1 =$ ☐. $8 \times \frac{5}{5} = 8 \times$ ☐ $=$ ☐.

$9 \times \frac{10}{10} = 9 \times$ ☐ $=$ ☐. $12 \times \frac{4}{4} = 12 \times$ ☐ $=$ ☐.

$\frac{15}{15} \times 3 =$ ☐ $\times 3 =$ ☐. $\frac{100}{100} \times 2 =$ ☐ $\times 2 =$ ☐.

Millions



To read this number, say: 613 million, 438 thousand, 127.

Fill in the missing numbers or words to show how each number below is read.

527, 613, 145 =
[ ] million, [ ] thousand, [ ].

292, 062, 037 =
292 [ ], 62 [ ], 37.

538 = 538 ones.
538,000 = 538 [ ].
538,000,000 = 538 [ ].

Write each number in numerals.

216 million, 125 thousand, 248
= [ ], [ ], [ ].

459 million, 24 thousand, 1
= [ ], [ ], [ ].

Count by hundred thousands.

100,000				

10 hundred thousands = 1 million.

20 hundred thousands = [ ] million.

Half of a million = [ ] hundred thousands.

600,000 = (more less) than half a million.

400,000 = (more less) than half a million.

4,500,000 is halfway between
4 million and [ ] million.

16,500,000 is halfway between [ ]
million and [ ] million.

8,300,000 is nearer to [ ] million than to 9 million.

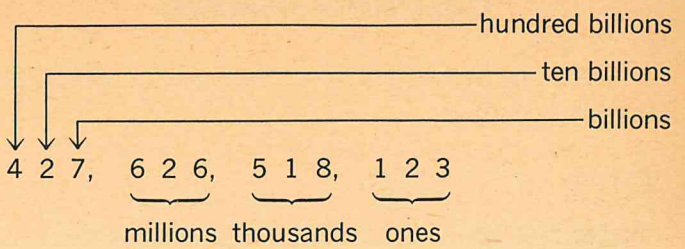
12,700,000 is nearer to [ ] million than to 12 million.

125,299,999 is about [ ] million.

56,801,000 is about [ ] million.

7,499,999 is about [ ] million.

Billions



To read this number, say: 427 billion, 626 million, 518 thousand, 123.

Fill in the missing numbers or words to show how each number below is read.

189, 513, 475, 187 =
[ ] billion, [ ] million,
[ ] thousand, [ ].

251, 803, 048, 071 =
251 [ ], 803 [ ],
48 [ ], 71.

624,000 = 624 [ ].
624,000,000 = 624 [ ].
624,000,000,000 = 624 [ ].

Write each number in numerals.

372 billion, 462 million, 175 thousand
= [ ], [ ], [ ], [ ].

456 billion, 235 thousand, 429
= [ ], [ ], [ ], [ ].

Count by hundred millions.

100,000,000			

10 hundred million = 1 billion.

30 hundred millions = [ ] billions.

Half of a billion = [ ] hundred million.

300,000,000 = (more less) than half a billion.

700,000,000 = (more less) than half a billion.

6,200,000,000 is about [ ] billion.

4,700,000,000 is about [ ] billion.

19,800,000,000 is about [ ] billion.

Division without a Pyramid Answer

From now on we shall use a new way of writing the answer in division.

Example Divide 456 by 8.

$$\begin{array}{r} 8 \\ \times 50 \\ \hline 400 \end{array} \qquad \begin{array}{r} 8 \\ \times 60 \\ \hline 480 \end{array}$$

Estimate: about 50.

Old Way

$$\begin{array}{r} 50 \\ 8\overline{)456} \end{array}$$

New Way

$$\begin{array}{r} 5 \\ 8\overline{)456} \end{array}$$

Instead of writing 50 in the quotient, write 5 in the tens place. This is just as good, because a 5 in the tens place stands for 50. Here is how you finish the exercise in both ways:

$$\begin{array}{r} 8 \\ \times 7 \\ \hline 56 \end{array}$$

$$\begin{array}{r} 7 \\ 50 \\ 8\overline{)456} \\ 400 \\ \hline 56 \\ 56 \\ \hline \end{array}$$

$$\begin{array}{r} 57 \\ 8\overline{)456} \\ 400 \\ \hline 56 \\ 56 \\ \hline \end{array}$$

Example Divide 4368 by 7.

$$\begin{array}{r} 7 \\ \times 600 \\ \hline 4200 \end{array} \qquad \begin{array}{r} 7 \\ \times 700 \\ \hline 4900 \end{array}$$

Estimate: about 600. Write 6 in the hundreds place:

$$\begin{array}{r} 6 \\ 7\overline{)4368} \\ 4200 \\ \hline 168 \end{array}$$

Now you divide 168 by 7:

$20 \times 7 = 140.$ $30 \times 7 = 210.$

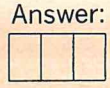
Estimate: about 20. Write 2 in the tens place:

$$\begin{array}{r} 62 \\ 7\overline{)4368} \\ 4200 \\ \hline 168 \\ 140 \\ \hline 28 \\ 28 \\ \hline \end{array}$$

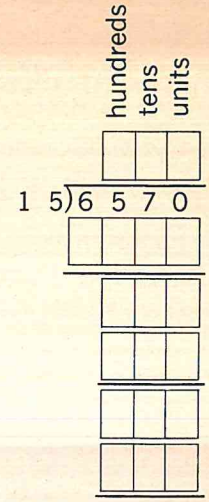
$28 \div 7 = 4.$

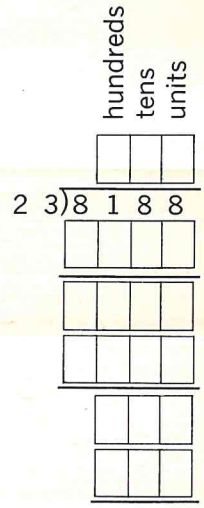
Write 4 in the units place.

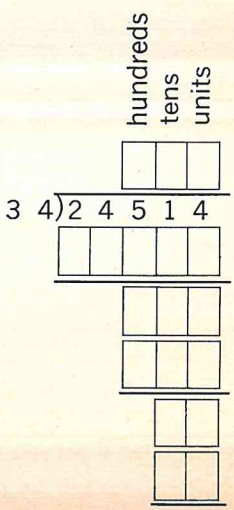
Answer:



Divide, and write the answer in the new way.







Zero in the Quotient

Divide 1827 by 9.

$$\begin{array}{r} 9 \\ \times 200 \\ \hline 1800 \end{array} \qquad \begin{array}{r} 3 \\ \times 9 \\ \hline 27 \end{array}$$

Old Way

$$\begin{array}{r} \quad 2\,0\,0\,3 \\ 9\overline{)1\,8\,2\,7} \\ \underline{1\,8\,0\,0} \\ \quad 2\,7 \\ \quad \underline{2\,7} \end{array}$$

New Way — Place holder

$$\begin{array}{r} \quad 2\,0\,3 \\ 9\overline{)1\,8\,2\,7} \\ \underline{1\,8\,0\,0} \\ \quad 2\,7 \\ \quad \underline{2\,7} \end{array}$$

In the new way, you show 200 by writing 2 in the hundreds place. You show 3 by writing 3 in the units place. To hold the tens place, write 0 in it. The quotient is 203.

Divide 21,280 by 7.

$3000 \times 7 = 21{,}000.$

$40 \times 7 = 280.$

Old Way

$$\begin{array}{r} \quad 4\,0 \\ \quad 3\,0\,0\,0 \\ 7\overline{)2\,1\,2\,8\,0} \\ \underline{2\,1\,0\,0\,0} \\ \quad 2\,8\,0 \\ \quad \underline{2\,8\,0} \end{array}$$

New Way — Place holders

$$\begin{array}{r} \quad 3\,0\,4\,0 \\ 7\overline{)2\,1\,2\,8\,0} \\ \underline{2\,1\,0\,0\,0} \\ \quad 2\,8\,0 \\ \quad \underline{2\,8\,0} \end{array}$$

In the new way, you show 3000 by writing 3 in the thousands place. You show 40 by writing 4 in the tens place. To hold the hundreds place and the units place, write 0's in them. The quotient is 3040.

Whenever there are empty places after the first digit of a quotient, put zeros in to hold these places.

Divide 4575 by 15.

$300 \times 15 = 4500.$

$5 \times 15 = 75.$

Old Way

$$\begin{array}{r} \quad 3\,0\,5 \\ 1\,5\overline{)4\,5\,7\,5} \\ \underline{4\,5\,0\,0} \\ \quad 7\,5 \\ \quad \underline{7\,5} \end{array}$$

New Way — Place holder

$$\begin{array}{r} \quad 3\,0\,5 \\ 1\,5\overline{)4\,5\,7\,5} \\ \underline{4\,5\,0\,0} \\ \quad 7\,5 \\ \quad \underline{7\,5} \end{array}$$

Write out each division example in two ways. Use 0 as a place holder wherever it is needed.

Old Way

$$1\,2\overline{)6\,0\,4\,8}$$

New Way

$$1\,2\overline{)6\,0\,4\,8}$$

Old Way

$$6\overline{)1\,8\,2\,5\,2}$$

New Way

$$6\overline{)1\,8\,2\,5\,2}$$

Old Way

$$1\,7\overline{)1\,0\,2\,8\,5}$$

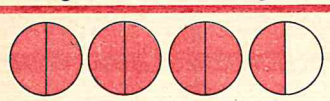
New Way

$$1\,7\overline{)1\,0\,2\,8\,5}$$

A Whole Number Times a Unit Fraction

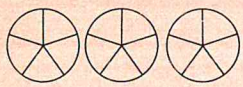
To find $6 \times \frac{1}{3}$, color 6 thirds.

$6 \times \frac{1}{3} = 6$ thirds $= \frac{6}{3} = 2$ wholes $= 2$.



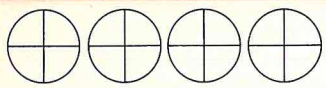
To find $7 \times \frac{1}{2}$, color 7 halves.

$7 \times \frac{1}{2} = 7$ halves $= \frac{7}{2} = \square\frac{\square}{\square}$.



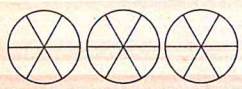
To find $10 \times \frac{1}{5}$, color $\square$ fifths.

$10 \times \frac{1}{5} = \square$ fifths $= \frac{\square}{5} = \square$.



To find $9 \times \frac{1}{4}$, color $\square$ fourths.

$9 \times \frac{1}{4} = \square$ fourths $= \frac{\square}{4} = \square\frac{\square}{\square}$.



To find $8 \times \frac{1}{6}$, color $\square$ sixths.

$8 \times \frac{1}{6} = \square$ sixths $= \frac{\square}{6} = \square\frac{\square}{6} = \square\frac{\square}{\square}$.

$5 \times \frac{1}{3} = \square$ thirds $= \frac{\square}{\square} = \square\frac{\square}{\square}$.

$8 \times \frac{1}{5} = \square$ fifths $= \frac{\square}{\square} = \square\frac{\square}{\square}$.

$4 \times \frac{1}{4} = \square$ fourths $= \frac{\square}{\square} = \square$.

$6 \times \frac{1}{10} = \square$ tenths $= \frac{\square}{\square} = \frac{\square}{\square}$.

$9 \times \frac{1}{3} = \square$ thirds $= \frac{\square}{\square} = \square$.

$8 \times \frac{1}{2} = \frac{\square}{\square} = \square$.

$10 \times \frac{1}{9} = \frac{\square}{\square} = \square\frac{\square}{\square}$.

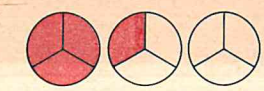
$10 \times \frac{1}{3} = \frac{\square}{\square} = \square\frac{\square}{\square}$.

$12 \times \frac{1}{4} = \frac{\square}{\square} = \square$.

$3 \times \frac{1}{6} = \frac{\square}{\square} = \frac{\square}{\square}$.

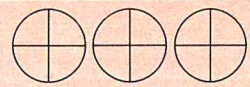
$5 \times \frac{1}{2} = \frac{\square}{\square} = \square\frac{\square}{\square}$.

A Whole Number Times Any Fraction



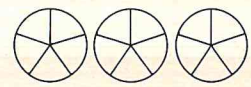
To find $2 \times \frac{2}{3}$, color 2 sets of 2 thirds each.

$2 \times \frac{2}{3} = \frac{4}{3} = \square\frac{\square}{\square}$.



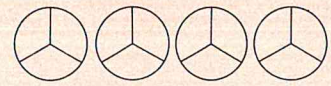
To find $4 \times \frac{3}{4}$, color 4 sets of 3 fourths each.

$4 \times \frac{3}{4} = \frac{\square}{4} = \square$.



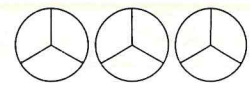
To find $3 \times \frac{2}{5}$, color $\square$ sets of 2 fifths each.

$3 \times \frac{2}{5} = \frac{\square}{5} = \square\frac{\square}{\square}$.



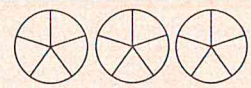
To find $5 \times \frac{2}{3}$, color $\square$ sets of $\square$ thirds each.

$5 \times \frac{2}{3} = \frac{\square}{\square} = \square\frac{\square}{\square}$.



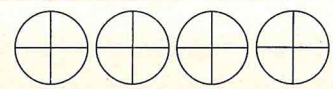
To find $3 \times \frac{2}{3}$, color $\square$ sets of $\square$ thirds each.

$3 \times \frac{2}{3} = \frac{\square}{\square} = \square$.



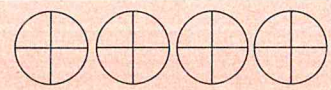
To find $4 \times \frac{2}{5}$, color $\square$ sets of $\square$ fifths each.

$4 \times \frac{2}{5} = \frac{\square}{\square} = \square\frac{\square}{\square}$.



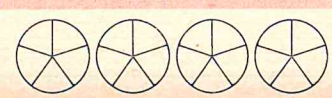
To find $5 \times \frac{3}{4}$, color $\square$ sets of $\square$ fourths each.

$5 \times \frac{3}{4} = \frac{\square}{\square} = \square\frac{\square}{\square}$.



To find $6 \times \frac{2}{4}$, color $\square$ sets of $\square$ fourths each.

$6 \times \frac{2}{4} = \frac{\square}{\square} = \square$.



To find $2 \times \frac{4}{5}$, color $\square$ sets of $\square$ fifths each.

$2 \times \frac{4}{5} = \frac{\square}{\square} = \square\frac{\square}{\square}$.

A Whole Number Times Any Fraction

Find $2 \times \frac{3}{5}$. Instead of drawing a diagram, use the associative law of multiplication to think out the product.

$$2 \times \frac{3}{5} = 2 \times \left(3 \times \frac{1}{5}\right)$$

$$= (2 \times 3) \times \frac{1}{5} = 6 \times \frac{1}{5} = \frac{6}{5} = \square\frac{\square}{\square}.$$

$$4 \times \frac{2}{9} = 4 \times \left(\square \times \frac{1}{9}\right)$$

$$= \left(4 \times \square\right) \times \frac{1}{9} = \square \times \frac{1}{9} = \frac{\square}{9}.$$

$$3 \times \frac{3}{4} = 3 \times \left(\square \times \frac{1}{4}\right)$$

$$= \left(3 \times \square\right) \times \frac{1}{4} = \square \times \frac{1}{4} = \frac{\square}{4} = \square\frac{\square}{\square}.$$

$$5 \times \frac{2}{5} = 5 \times \left(\square \times \frac{1}{5}\right)$$

$$= \left(5 \times \square\right) \times \frac{1}{5} = \square \times \frac{1}{5} = \frac{\square}{5} = \square.$$

$$6 \times \frac{3}{4} = \square \times \left(\square \times \frac{1}{4}\right)$$

$$= \left(\square \times \square\right) \times \frac{1}{4} = \square \times \frac{1}{4} = \frac{\square}{4} = \square\frac{\square}{\square}.$$

$$2 \times \frac{4}{5} = \square \times \left(\square \times \frac{1}{5}\right)$$

$$= \left(\square \times \square\right) \times \frac{1}{5} = \square \times \frac{1}{5} = \frac{\square}{5} = \square\frac{\square}{\square}.$$

$$6 \times \frac{3}{2} = \square \times \left(\square \times \frac{1}{2}\right)$$

$$= \left(\square \times \square\right) \times \frac{1}{2} = \square \times \frac{1}{2} = \frac{\square}{2} = \square.$$

$$5 \times \frac{2}{3} = \square \times \left(\square \times \frac{1}{3}\right)$$

$$= \left(\square \times \square\right) \times \frac{1}{3} = \square \times \frac{1}{3} = \frac{\square}{3} = \square\frac{\square}{\square}.$$

$$10 \times \frac{3}{5} = \square \times \left(\square \times \frac{1}{5}\right)$$

$$= \left(\square \times \square\right) \times \frac{1}{5} = \square \times \frac{1}{5} = \frac{\square}{5} = \square.$$

Rule: To multiply a fraction by a whole number, multiply the numerator of the fraction by the whole number, and keep the same denominator.

Example $6 \times \frac{2}{3} = \frac{6 \times 2}{3} = \frac{12}{3} = 4.$

$$5 \times \frac{3}{4} = \frac{5 \times 3}{4} = \frac{\square}{4} = \square\frac{\square}{\square}.$$

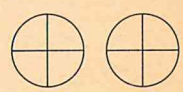
$4 \times \frac{2}{5} = \frac{8}{5} = \square\frac{\square}{\square}.$ $\quad$ $3 \times \frac{2}{9} = \frac{6}{9} = \frac{\square}{\square}.$

Use the rule to find each of these products. Check your answer by coloring sets of parts, as you did on page 66.

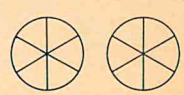
$4 \times \frac{2}{3} = \frac{\square}{3} = \square\frac{\square}{\square}.$

$3 \times \frac{2}{5} = \frac{\square}{5} = \square\frac{\square}{\square}.$

$2 \times \frac{3}{4} = \frac{\square}{4} = \square\frac{\square}{\square} = \square\frac{\square}{\square}.$



$2 \times \frac{5}{6} = \frac{\square}{6} = \square\frac{\square}{\square} = \square\frac{\square}{\square}.$

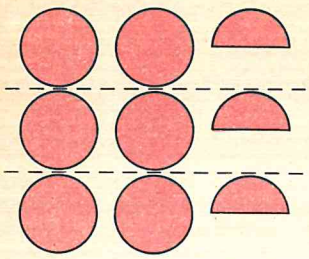


Use the rule to find these products.

$3 \times \frac{4}{10} =$	$4 \times \frac{4}{5} =$
$5 \times \frac{2}{7} =$	$2 \times \frac{25}{100} =$
$8 \times \frac{2}{9} =$	$5 \times \frac{2}{10} =$
$4 \times \frac{3}{4} =$	$8 \times \frac{5}{10} =$
$10 \times \frac{3}{100} =$	$6 \times \frac{2}{3} =$
$6 \times \frac{2}{10} =$	$5 \times \frac{5}{6} =$
$5 \times \frac{2}{5} =$	$2 \times \frac{3}{7} =$
$8 \times \frac{2}{3} =$	$2 \times \frac{3}{4} =$
$7 \times \frac{1}{2} =$	$6 \times \frac{2}{5} =$

A Whole Number Times a Mixed Number

First Method Use addition.



$3 \times 2\frac{1}{2}$:

$$2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} = \square\frac{\square}{\square} = \square\frac{\square}{\square}.$$

$3 \times 2\frac{1}{3}$:

$$2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} = \square\frac{\square}{\square} = \square.$$

$3 \times 4\frac{2}{5}$:

$$4\frac{2}{5} + 4\frac{2}{5} + 4\frac{2}{5} = \square\frac{\square}{\square} = \square\frac{\square}{\square}.$$

$2 \times 5\frac{1}{4}$:

$$5\frac{1}{4} + 5\frac{1}{4} = \square\frac{\square}{\square} = \square\frac{\square}{\square}.$$

$2 \times 3\frac{1}{6}$:

$$3\frac{1}{6} + 3\frac{1}{6} = \square\frac{\square}{\square} = \square\frac{\square}{\square}.$$

$4 \times 2\frac{2}{3}$:

$$2\frac{2}{3} + 2\frac{2}{3} + 2\frac{2}{3} + 2\frac{2}{3} = \square\frac{\square}{\square} = \square\frac{\square}{\square}.$$

$4 \times 7\frac{2}{5}$:

$$7\frac{2}{5} + 7\frac{2}{5} + 7\frac{2}{5} + 7\frac{2}{5} = \square\frac{\square}{\square} = \square\frac{\square}{\square}.$$

$$3\frac{1}{5} + 3\frac{1}{5} = \square\frac{\square}{\square} \qquad 3\frac{1}{5} \times 2 = \square\frac{\square}{\square}$$

$$5\frac{2}{7} + 5\frac{2}{7} = \square\frac{\square}{\square} \qquad 5\frac{2}{7} \times 2 = \square\frac{\square}{\square}$$

Second Method Change the mixed number to an improper fraction.

$$3 \times 2\frac{1}{2} = 3 \times \frac{5}{2} = \frac{\square}{2} = \square\frac{\square}{\square}.$$

$$4 \times 1\frac{3}{4} = 4 \times \frac{\square}{4} = \frac{\square}{4} = \square.$$

$$2 \times 5\frac{1}{3} = 2 \times \frac{\square}{3} = \frac{\square}{3} = \square\frac{\square}{\square}.$$

$$3 \times 2\frac{2}{5} = 3 \times \frac{\square}{5} = \frac{\square}{5} = \square\frac{\square}{\square}.$$

$2 \times 2\frac{1}{3} =$

$3 \times 4\frac{3}{5} =$

Third Method Use the distributive law.

$$3 \times 2\frac{2}{3} = 3 \times (2 + \tfrac{2}{3}) = (3 \times 2) + (3 \times \tfrac{2}{3}) = \square + \frac{\square}{3} = \square + \square = \square.$$

$$4 \times 1\frac{2}{5} = 4 \times (1 + \tfrac{2}{5}) = (4 \times 1) + (4 \times \tfrac{2}{5}) = \square + \frac{\square}{5} = \square + \square\frac{\square}{5} = \square\frac{\square}{5}.$$

$$6 \times 3\frac{1}{2} = 6 \times \left(\square + \frac{\square}{\square}\right) = \left(6 \times \square\right) + \left(6 \times \frac{\square}{\square}\right) = \square + \frac{\square}{\square} = \square + \square = \square.$$

$$2 \times 5\frac{1}{3} = 2 \times \left(\square + \frac{\square}{\square}\right) = \left(2 \times \square\right) + \left(2 \times \frac{\square}{\square}\right) = \square + \frac{\square}{\square} = \square\frac{\square}{\square}.$$

$5 \times 2\frac{1}{4}$

$= 5 \times \left(\square + \frac{\square}{\square}\right)$

$=$

$3 \times 2\frac{3}{8}$

$= 3 \times \left(\square + \frac{\square}{\square}\right)$

$=$

A Whole Number Times a Mixed Number

Estimate the answer.

Find $4 \times 2\frac{2}{3}$.

$2\frac{2}{3}$ is more than 2, but less than 3. So $4 \times 2\frac{2}{3}$ is more than 4×2, or 8, and less than 4×3, or 12.

To compute the answer by the distributive law, multiply 2 by 4 and multiply $\frac{2}{3}$ by 4, and then add the products. We can arrange the work like this:

Third Method

$$4 \times \tfrac{2}{3} = \tfrac{8}{3} = 2\tfrac{2}{3}$$
$$4 \times 2 = 8$$
$$10\tfrac{2}{3}$$

$$2\tfrac{2}{3} \times 4$$

First estimate each product. Then find the exact answer by the third method.

$5 \times 6\frac{2}{3}$

Estimate:
more than 5×6, or ☐
less than 5×7, or ☐

$6\frac{2}{3} \times 5$

$5 \times \frac{2}{3} =$ ☐

$5 \times 6 =$ ☐

$8 \times 2\frac{1}{4}$

Estimate:
more than $8 \times$ ☐, or ☐
less than $8 \times$ ☐, or ☐

$2\frac{1}{4} \times 8$

$8 \times \frac{1}{4} =$ ☐

$8 \times 2 =$ ☐

$4 \times 3\frac{1}{5}$

Estimate:
more than $4 \times$ ☐, or ☐
less than $4 \times$ ☐, or ☐

$3\frac{1}{5} \times 4$

$4 \times \frac{1}{5} =$ ☐

$4 \times 3 =$ ☐

First estimate each product. Then find the exact answer by the third method.

$3 \times 12\frac{2}{3}$

Estimate:
more than ☐
less than ☐

$12\frac{2}{3} \times 3$

$4 \times 15\frac{1}{8}$

Estimate:
more than ☐
less than ☐

$15\frac{1}{8} \times 4$

$5 \times 10\frac{1}{2}$

Estimate:
more than ☐
less than ☐

$10\frac{1}{2} \times 5$

$8 \times 3\frac{2}{10}$

Estimate:
more than ☐
less than ☐

$3\frac{2}{10} \times 8$

$6 \times 2\frac{2}{5}$

Estimate:
more than ☐
less than ☐

$2\frac{2}{5} \times 6$

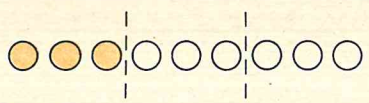
$7 \times 4\frac{3}{5}$

Estimate:
more than ☐
less than ☐

$4\frac{3}{5} \times 7$

Fractional Parts of Groups

$\frac{1}{2}$ of 6 means
divide 6 into 2 equal parts,
and take 1 part.
$\frac{1}{2}$ of 6 = ☐.

$\frac{1}{3}$ of 9 means 
divide 9 into 3 equal parts,
and take 1 part.
$\frac{1}{3}$ of 9 = ☐.

$\frac{1}{4}$ of 8 means
divide 8 into ☐ equal parts,
and take ☐ part.
$\frac{1}{4}$ of 8 = ☐.

Divide these 6 stars into 3 equal groups, and color 1 group.
$\frac{1}{3}$ of 6 = ☐.

Divide these 8 blocks into 2 equal groups, and color 1 group.
$\frac{1}{2}$ of 8 = ☐.

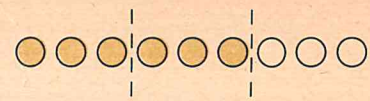
○○○○○○○○○○ $\frac{1}{2}$ of 10 = ☐.

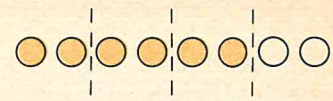
☐☐☐☐☐☐☐☐☐☐☐☐ $\frac{1}{4}$ of 12 = ☐.

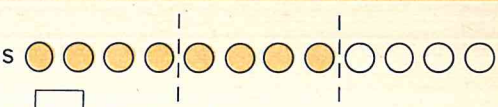
$\frac{1}{2}$ of 6 = 6 ÷ 2 = ☐.
$\frac{1}{3}$ of 9 = 9 ÷ 3 = ☐.
$\frac{1}{4}$ of 8 = 8 ÷ 4 = ☐.
$\frac{1}{3}$ of 6 = 6 ÷ 3 = ☐.
$\frac{1}{3}$ of 15 = 15 ÷ 3 = ☐.
$\frac{1}{4}$ of 16 = 16 ÷ 4 = ☐.
$\frac{1}{5}$ of 10 = ☐ ÷ ☐ = ☐.
$\frac{1}{6}$ of 12 = ☐ ÷ ☐ = ☐.

○○○○○ $\frac{1}{2}$ of 5 = 5 ÷ 2 = $2\frac{1}{2}$.

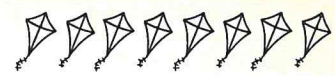
$\frac{1}{3}$ of 7 = ☐ ÷ ☐ = ☐ $\frac{\square}{\square}$.
$\frac{1}{4}$ of 10 = ☐ ÷ ☐ = ☐ $\frac{\square}{\square}$.
$\frac{1}{5}$ of 6 = ☐ ÷ ☐ = ☐ $\frac{\square}{\square}$.
$\frac{1}{2}$ of 9 = ☐ ÷ ☐ = ☐ $\frac{\square}{\square}$.
$\frac{1}{3}$ of 5 = ☐ ÷ ☐ = ☐ $\frac{\square}{\square}$.
$\frac{1}{4}$ of 5 = ☐ ÷ ☐ = ☐ $\frac{\square}{\square}$.

$\frac{2}{3}$ of 9 means 
divide 9 into 3 equal parts,
and take 2 parts.
$\frac{2}{3}$ of 9 = ☐.

$\frac{3}{4}$ of 8 means 
divide 8 into ☐ equal parts,
and take ☐ parts.
$\frac{3}{4}$ of 8 = ☐.

$\frac{2}{3}$ of 12 means 
divide 12 into ☐ equal parts,
and take ☐ parts.
$\frac{2}{3}$ of 12 = ☐.

Divide these 6 tops into 3 equal groups, and color 2 groups.
$\frac{2}{3}$ of 6 = ☐.

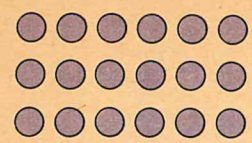
Divide these 8 kites into 4 equal groups, and color 2 groups. 
$\frac{2}{4}$ of 8 = ☐.

○○○○○○○○○○ $\frac{2}{5}$ of 10 = ☐.

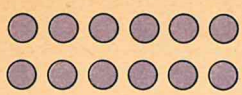
☐☐☐☐☐☐☐☐☐☐☐☐ $\frac{3}{4}$ of 12 = ☐.

$\frac{2}{3}$ of 9 = 2 × ($\frac{1}{3}$ of 9) = 2 × 3 = ☐.
$\frac{3}{4}$ of 8 = 3 × ($\frac{1}{4}$ of 8) = 3 × 2 = ☐.
$\frac{2}{3}$ of 12 = 2 × ($\frac{1}{\square}$ of 12) = 2 × ☐ = ☐.
$\frac{2}{3}$ of 6 = 2 × ($\frac{1}{\square}$ of 6) = 2 × ☐ = ☐.
$\frac{2}{4}$ of 8 = 2 × ($\frac{1}{\square}$ of 8) = 2 × ☐ = ☐.
$\frac{2}{5}$ of 10 = 2 × ($\frac{1}{\square}$ of 10) = 2 × ☐ = ☐.
$\frac{3}{4}$ of 12 = ☐ × ($\frac{1}{\square}$ of 12) = ☐ × ☐ = ☐.
$\frac{4}{5}$ of 10 = ☐ × ($\frac{1}{\square}$ of 10) = ☐ × ☐ = ☐.
$\frac{3}{4}$ of 16 = ☐ × ($\frac{1}{\square}$ of 16) = ☐ × ☐ = ☐.

A Unit Fraction Times a Whole Number



3×6 means
3 times as much as 6.
$3 \times 6 = \square$.



2×6 means
2 times as much as 6.
$2 \times 6 = \square$.

1×6 means
1 times as much as 6.
$1 \times 6 = \square$.

$\frac{1}{2} \times 6$ means
one half as much as 6.
$\frac{1}{2} \times 6 = \square$.

$\frac{1}{3}$ times 6 means
one third as much as 6.
$\frac{1}{3} \times 6 = \square$.

First Method

Think: When you multiply by a fraction, times means of.

$\frac{1}{2} \times 8 = \frac{1}{2}$ of $8 = 8 \div \square = \square$.

$\frac{1}{3} \times 9 = \frac{1}{3}$ of $9 = 9 \div \square = \square$.

$\frac{1}{5} \times 10 = \frac{1}{5}$ of $10 = 10 \div \square = \square$.

Second Method

Use the commutative law of multiplication.

$\frac{1}{3} \times 12 = 12 \times \frac{1}{3} = \frac{12 \times 1}{3} = \frac{\square}{3} = \square$.

$\frac{1}{4} \times 8 = 8 \times \frac{1}{4} = \frac{8 \times 1}{4} = \frac{\square}{4} = \square$.

$\frac{1}{2} \times 4 = 4 \times \frac{1}{2} = \frac{\square \times 1}{2} = \frac{\square}{2} = \square$.

$\frac{1}{3} \times 6 = \square \times \frac{1}{3} = \frac{\square \times 1}{3} = \frac{\square}{3} = \square$.

$\frac{1}{5} \times 10 = \square \times \frac{1}{5} = \frac{\square \times 1}{5} = \frac{\square}{5} = \square$.

$\frac{1}{3} \times 15 = \square \times \frac{1}{3} = \frac{\square \times \square}{\square} = \frac{\square}{\square} = \square$.

A Fraction Times a Whole Number

First Method

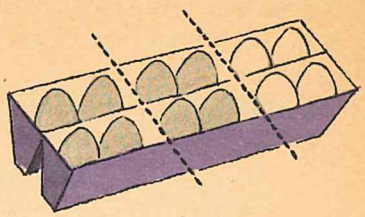
Think: When you multiply by a fraction, times means of.

$\frac{2}{3} \times 12$

$= \frac{2}{3}$ of 12

$= 2 \times \left(\frac{1}{3} \text{ of } 12\right)$

$= 2 \times \square = \square$.



$\frac{3}{4} \times 8 = \frac{3}{4}$ of $8 = 3 \times \left(\frac{1}{4} \text{ of } 8\right) = 3 \times \square = \square$.

$\frac{2}{5} \times 10 = \frac{2}{5}$ of $10 = 2 \times \left(\frac{1}{5} \text{ of } 10\right) = 2 \times \square = \square$.

Second Method

Use the associative law of multiplication.

$\frac{2}{3} \times 9 = \left(2 \times \frac{1}{3}\right) \times 9$

$= 2 \times \left(\frac{1}{3} \times 9\right)$

$= 2 \times \square = \square$.

$\frac{3}{5} \times 10 = \left(3 \times \frac{1}{5}\right) \times 10$

$= 3 \times \left(\frac{1}{5} \times 10\right)$

$= 3 \times \square = \square$.

$\frac{3}{2} \times 4 = \left(3 \times \frac{1}{2}\right) \times 4$

$= 3 \times \left(\frac{\square}{\square} \times \square\right)$

$= 3 \times \square = \square$.

$\frac{4}{3} \times 6 = \left(4 \times \frac{1}{3}\right) \times 6$

$= 4 \times \left(\frac{\square}{\square} \times \square\right)$

$= 4 \times \square = \square$.

Third Method

Use the commutative law of multiplication.

$\frac{3}{4} \times 4 = 4 \times \frac{3}{4} = \frac{\square \times \square}{4} = \frac{\square}{4} = \square$.

$\frac{2}{3} \times 6 = 6 \times \frac{2}{3} = \frac{\square \times \square}{3} = \frac{\square}{3} = \square$.

$\frac{3}{5} \times 10 = \square \times \frac{3}{5} = \frac{\square \times \square}{5} = \frac{\square}{5} = \square$.

$\frac{5}{2} \times 4 = \square \times \frac{5}{2} = \frac{\square \times \square}{2} = \frac{\square}{2} = \square$.

$\frac{3}{2} \times 8 = \square \times \frac{\square}{\square} = \frac{\square \times \square}{\square} = \frac{\square}{\square} = \square$.

$\frac{2}{5} \times 10 = \square \times \frac{\square}{\square} = \frac{\square \times \square}{\square} = \frac{\square}{\square} = \square$.

A Fraction Times a Whole Number

$\frac{2}{3} \times 15 = 15 \times \frac{2}{3} = \frac{15 \times 2}{3} = \frac{30}{3} = 10.$

The step in the colored box suggests another way of multiplying a fraction times a whole number.

Fourth Method — Multiply the numerator of the fraction by the whole number, and keep the same denominator.

$\frac{2}{3} \times 30 = \frac{30 \times 2}{3} = \frac{2 \times 30}{3} = \frac{\square}{3} = \square.$

$\frac{3}{5} \times 20 = \frac{3 \times 20}{5} = \frac{\square}{5} = \square.$ $\frac{3}{4} \times 12 = \frac{3 \times 12}{4} = \frac{\square}{4} = \square.$

$\frac{2}{5} \times 10 = \frac{\square}{5} = \square.$

$\frac{2}{3} \times 9 = \frac{\square}{3} = \square.$

$\frac{3}{4} \times 4 = \frac{\square}{4} = \square.$

$\frac{4}{5} \times 10 = \frac{\square}{5} = \square.$

$\frac{2}{3} \times 18 =$

$\frac{5}{2} \times 4 =$

$\frac{3}{2} \times 6 =$

$\frac{5}{3} \times 6 =$

$\frac{2}{3} \times 5 =$

$\frac{3}{2} \times 5 =$

A Mixed Number Times a Whole Number

Multiply $3\frac{1}{2} \times 8$.

$3\frac{1}{2} \times 8 = 8 \times 3\frac{1}{2}$ — by the commutative law of multiplication.

$= 8 \times (3 + \frac{1}{2})$

$= (8 \times 3) + (8 \times \frac{1}{2})$ — by the distributive law.

$= (3 \times 8) + (\frac{1}{2} \times 8)$ — because $8 \times 3 = 3 \times 8$ and $8 \times \frac{1}{2} = \frac{1}{2} \times 8$.

So $3\frac{1}{2} \times 8 = (3 \times 8) + (\frac{1}{2} \times 8) = 24 + 4 = \square.$

$5\frac{1}{3} \times 6 = (5 \times 6) + (\frac{1}{3} \times 6) = \square + \square = \square.$

$4\frac{1}{4} \times 8 = (4 \times 8) + (\frac{1}{4} \times 8) = \square + \square = \square.$

$5\frac{2}{3} \times 6 = (5 \times 6) + (\frac{2}{3} \times 6) = \square + \square = \square.$

$4\frac{1}{2} \times 10 = (4 \times 10) + (\frac{1}{2} \times 10) = \square + \square = \square.$

$4\frac{1}{3} \times 6 = (4 \times 6) + (\frac{1}{3} \times 6) = 24 + 2 = 26.$

The work can also be arranged in this way:

Estimate:
more than $4 \times 6 = 24$
less than $5 \times 6 = 30$

$\frac{1}{3} \times 6 = 2$
$4 \times 6 = 24$
26

First estimate each product. Then find the exact product by the method shown above.

$6\frac{1}{2} \times 8$

Estimate:
more than $6 \times 8 = \square$
less than $7 \times 8 = \square$

$7\frac{2}{5} \times 10$

Estimate:
more than $7 \times 10 = \square$
less than $8 \times 10 = \square$

$8\frac{3}{4} \times 8$

more than $\square$
less than $\square$

$4\frac{1}{2} \times 24$

more than $\square$
less than $\square$

$5\frac{1}{3} \times 24$

more than $\square$
less than $\square$

Changing Improper Fractions to Decimals

To change an improper fraction to a decimal first change it to a mixed number.

Example $\frac{5}{2} = 2\frac{1}{2} = 2\frac{5}{10} = 2.5$.

Change each of these to a decimal, using tenths:

$\frac{6}{5} = \square\frac{\square}{5} = \square\frac{\square}{10} = \square.\square$.

$\frac{8}{5} = \square\frac{\square}{5} = \square\frac{\square}{10} = \square.\square$.

$\frac{14}{5} = \square\frac{\square}{5} = \square\frac{\square}{10} = \square.\square$.

$\frac{22}{5} =$

$\frac{33}{5} =$

$\frac{3}{2} =$

$\frac{7}{2} =$

$\frac{13}{10} = 1\frac{3}{10} = 1.3$.

$\frac{36}{10} = \square\frac{\square}{10} = \square.\square$.

$\frac{24}{10} =$

$\frac{48}{10} =$

Change each to a decimal, using hundredths.

$\frac{9}{2} = 4\frac{1}{2} = 4\frac{50}{100} = 4.50$.

$\frac{11}{2} = \square\frac{\square}{2} = \square\frac{\square}{100} = \square.\square\square$.

$\frac{17}{10} = \square\frac{\square}{10} = \square\frac{\square}{100} = \square.\square\square$.

$\frac{23}{5} = \square\frac{\square}{5} = \square\frac{\square}{100} = \square.\square\square$.

$\frac{9}{4} = \square\frac{\square}{4} = \square\frac{\square}{100} = \square.\square\square$.

$\frac{102}{100} = 1\frac{2}{100} = \square.\square\square$.

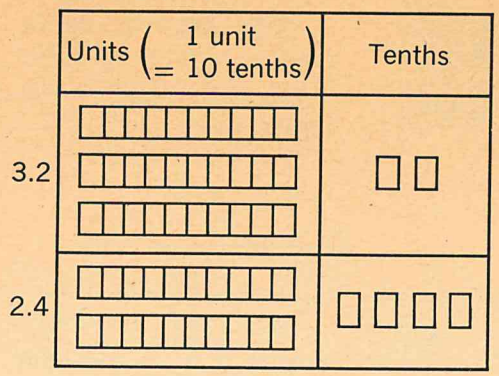
$\frac{315}{100} = \square\frac{\square}{100} = \square.\square\square$.

$\frac{205}{100} =$

$\frac{427}{100} =$

Adding Decimals

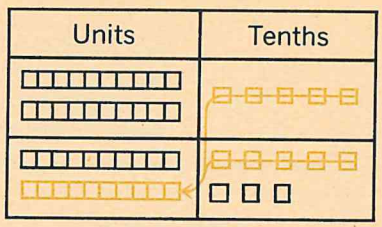
Add 3.2 + 2.4.



$$\begin{array}{r} 3.2 \\ +\ 2.4 \\ \hline \square.\square \end{array}$$

Add the tenths. Then add the units.

Add 2.5 + 1.8.



$$\begin{array}{r} {\scriptstyle 1} \\ 2.5 \\ +\ 1.8 \\ \hline \square.3 \end{array}$$

Exchange 10 tenths for 1 unit.

5 tenths + 8 tenths = 13 tenths
= 10 tenths + 3 tenths
= 1 unit + 3 tenths.

Add.

$$\begin{array}{r} 6.2 \\ +4.3 \\ \hline \square \end{array} \qquad \begin{array}{r} 7.9 \\ +1.6 \\ \hline \square \end{array} \qquad \begin{array}{r} 8.3 \\ +2.7 \\ \hline \square \end{array} \qquad \begin{array}{r} 12.6 \\ +4.8 \\ \hline \square \end{array}$$

Subtracting Decimals

$$\begin{array}{r} 8.7 \\ -\ 2.3 \\ \hline \square\square \end{array} \qquad \begin{array}{r} 6.9 \\ -\ 4.1 \\ \hline \square\square \end{array}$$

Subtract the tenths. Then subtract the units.

$$\begin{array}{r} {\scriptstyle 7\ \ 11} \\ \not{8}.\not{1} \\ -\ 3.4 \\ \hline \square.\square \end{array}$$

Exchange 1 unit for 10 tenths, to be able to subtract.

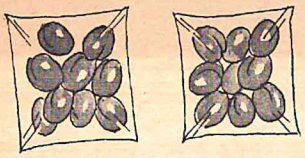
Subtract.

$$\begin{array}{r} 15.7 \\ -3.2 \\ \hline \square \end{array} \qquad \begin{array}{r} 9.4 \\ -1.5 \\ \hline \square \end{array} \qquad \begin{array}{r} 23.8 \\ -9.2 \\ \hline \square \end{array} \qquad \begin{array}{r} 37.2 \\ -5.8 \\ \hline \square \end{array}$$

Subtract. Then check, by subtracting in fraction form.

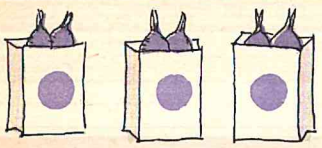
$$\begin{array}{rcl} 8.3 & = & 8\frac{\square}{10} = 7\frac{\square}{10} \\ -\ 1.6 & = & -1\frac{\square}{10} = -1\frac{\square}{10} \\ \hline \square.\square & & \square\frac{\square}{10} \end{array}$$

A Whole Number Times a Denominate Number



Each bag of plums weighs 1 lb. 2 oz. How much do both bags weigh together?

1 lb.	2 oz.		1 lb.	2 oz.
+1 lb.	2 oz.	or	×2	×2
2 lb.	4 oz.		2 lb.	4 oz.



Each bag of onions weighs 3 lb. 9 oz. How much do all three bags weigh together?

3 lb.	9 oz.			3 lb.	9 oz.	
3 lb.	9 oz.	or		×3	×3	
+3 lb.	9 oz.			9 lb.	27 oz.	
9 lb.	27 oz.	=		10 lb.	11 oz.	

Remember 16 oz. = 1 lb.

Mary tied 2 ft. 5 in. of ribbon around each of 3 packages. How much ribbon did she use?

2 ft.	5 in.	
×3	×3	
☐ ft.	☐ in.	
= ☐ ft.	☐ in.	

Each side of a square frame is 1 ft. 4 in. long. How much molding was used to make the frame?

1 ft.	4 in.	
×4	×4	
☐ ft.	☐ in.	
= ☐ ft.	☐ in.	

Remember 12 in. = 1 ft.

Multiply.

2 × (5 ft. 7 in.)
= ☐ ft. ☐ in.
= ☐ ft. ☐ in.

4 × (3 lb. 6 oz.)
= ☐ lb. ☐ oz.
= ☐ lb. ☐ oz.

5 × (2 lb. 7 oz.)
= ☐ lb. ☐ oz.
= ☐ lb. ☐ oz.

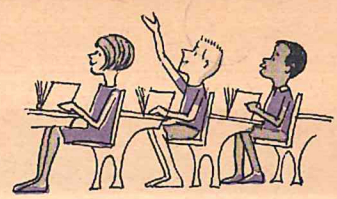
6 × (1 ft. 8 in.)
= ☐ ft. ☐ in.
= ☐ ft. ☐ in.

3 × (3 ft. 4 in.)
= ☐ ft. ☐ in.
= ☐ ft. ☐ in.

8 × (2 lb. 3 oz.)
= ☐ lb. ☐ oz.
= ☐ lb. ☐ oz.

A bus travels over its route one way in 1 hr. 45 min. How long does a round trip take?

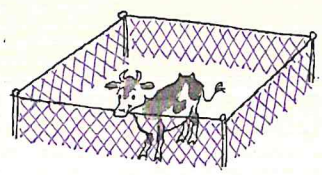
2 × (1 hr. 45 min.)
= ☐ hr. ☐ min.
= ☐ hr. ☐ min.



This class meets 3 times a week. Each session lasts 1 hr. 20 min. How much time is spent in class each week?

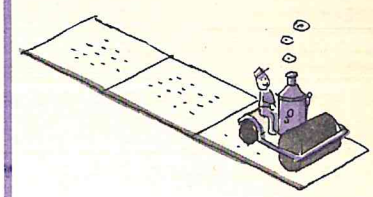
3 × (1 hr. 20 min.)
= ☐ hr. ☐ min.
= ☐ hr.

Remember 60 min. = 1 hr.



Each side of a square corral is 10 yd. 2 ft. long. How much fencing is used to enclose the corral?

4 × (10 yd. 2 ft.)
= ☐ yd. ☐ ft.
= ☐ yd. ☐ ft.



A concrete roadway was paved in 3 sections. Each section was 5 yd. 1 ft. long. How long were the 3 sections?

3 × (5 yd. 1 ft.)
= ☐ yd. ☐ ft.
= ☐ yd.

Remember 3 ft. = 1 yd.

Multiply.

4 × (2 hr. 15 min.)
= ☐ hr. ☐ min.
= ☐ hr.

2 × (7 yd. 2 ft.)
= ☐ yd. ☐ ft.
= ☐ yd. ☐ ft.

5 × (10 yd. 1 ft.)
= ☐ yd. ☐ ft.
= ☐ yd. ☐ ft.

6 × (3 hr. 30 min.)
= ☐ hr. ☐ min.
= ☐ hr.

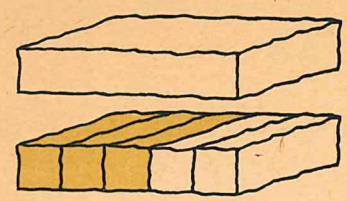
4 × (9 ft. 5 in.)
= ☐ ft. ☐ in.
= ☐ ft. ☐ in.

5 × (2 lb. 4 oz.)
= ☐ lb. ☐ oz.
= ☐ lb. ☐ oz.

5 × (8 hr. 20 min.)
= ☐ hr. ☐ min.
= ☐ hr. ☐ min.

6 × (7 ft. 2 in.)
= ☐ ft. ☐ in.
= ☐ ft.

What Part One Number Is of Another



A piece of fudge costs 5¢. What part of it will 3¢ buy? Imagine the fudge divided into 5 equal parts. Each part is worth 1¢. 3¢ will buy 3 parts. 3 parts out of 5 parts is the fraction $\frac{3}{5}$. So 3¢ will buy $\frac{3}{5}$ of the fudge.

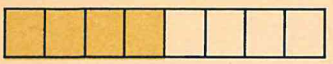
2 is what part of 7?

7 = 7 units. 2 = 2 units.

2 parts out of 7 parts is the fraction $\frac{2}{7}$.

So 2 is $\frac{2}{7}$ of 7.

Check. $\frac{2}{7}$ of $7 = \frac{2}{7} \times 7 = \frac{14}{7} = 2$.



4 is what part of 8?

8 = 8 units. 4 = 4 units.

4 parts out of 8 parts is the fraction $\frac{4}{8} = \frac{1}{2}$.

So 4 is $\frac{1}{2}$ of 8.

Check. $\frac{1}{2}$ of $8 = \frac{1}{2} \times 8 = \frac{8}{2} = 4$.

To find what part one number is of a second number, write a fraction that has the first number as numerator and the second number as denominator.

6 is what part of 7? 6 → is $\frac{\square}{\square}$ of 7.

8 is what part of 10? 8 → is $\frac{\square}{\square}$ of 10.

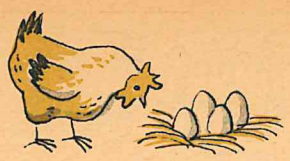
4 is what part of 6? 4 → is $\frac{\square}{\square}$ of 6.

9 is what part of 12? Answer: $\frac{\square}{\square} = \frac{\square}{\square}$.

10 is what part of 100? Answer: $\frac{\square}{\square} = \frac{\square}{\square}$.

50 is what part of 100? Answer: $\frac{\square}{\square} = \frac{\square}{\square}$.

30 is what part of 60? Answer: $\frac{\square}{\square} = \frac{\square}{\square}$.



If eggs cost 60¢ a dozen, how much do 4 eggs cost?

1 doz. = 12. 4 is $\frac{4}{12}$ of 12, or $\frac{1}{3}$ of 12.

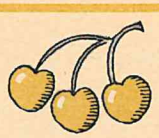
So 4 eggs cost $\frac{1}{3}$ of 60¢, or 20¢.

Mary does baby-sitting for 80¢ an hour. At this rate, how much is she paid for 30 minutes?

1 hr. = ☐ min.

30 min. = $\frac{30}{\square}$ of an hour, or $\frac{\square}{\square}$ of an hour.

She gets $\frac{\square}{\square}$ of 80¢, or ☐ ¢ for 30 minutes.



Cherries cost 48¢ a pound. How much do 4 oz. of cherries cost?

1 lb. = ☐ oz.

4 oz. = $\frac{\square}{\square}$ of a pound, or $\frac{\square}{\square}$ of a pound.

4 oz. cost $\frac{\square}{\square}$ of 48¢, or ☐ ¢.

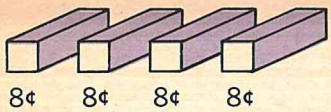
At 72¢ a dozen, how much do 6 eggs cost?

At 75¢ an hour, how much is Judy paid for 20 minutes of work?

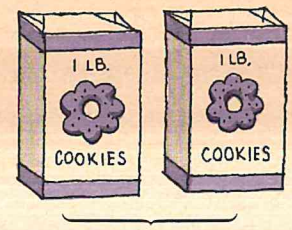
At 24¢ a pound, how much do 8 oz. of plums cost?

At 48¢ a dozen, how much do 3 cupcakes cost?

Finding the Whole When a Fractional Part Is Given



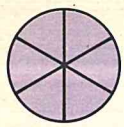
If $\frac{1}{4}$ lb. of margarine costs 8¢, how much does 1 lb. cost? 1 whole = 4 fourths. If 1 fourth costs 8¢, 4 fourths cost 4 × 8¢, or ☐¢.



If 2 lb. of cookies cost 80¢, how much does 1 lb. cost?

1 lb. costs 80¢ ÷ 2

or ☐¢.



If $\frac{1}{6}$ of a pie costs 15¢, how much does a whole pie cost?

1 whole = ☐ sixths.

1 whole pie costs ☐ × 15¢, or ☐¢.

$\frac{1}{3}$ of a number is 15. What is the number?

1 whole = ☐ thirds. ☐ × 15 = ☐.

$\frac{1}{4}$ of a number is 12. What is the number?

1 whole = ☐ fourths. ☐ × 12 = ☐.

$\frac{1}{5}$ of a number is 13. What is the number?

1 whole = ☐ fifths. ☐ × 13 = ☐.

$\frac{1}{2}$ of a number is 9. What is the number?

1 whole = ☐ halves. ☐ × 9 = ☐.

$\frac{1}{6}$ of a number is 23. What is the number?

1 whole = ☐ sixths. ☐ × 23 = ☐.

$\frac{1}{10}$ of a number is 14. What is the number?

1 whole = ☐ tenths. ☐ × 14 = ☐.

$\frac{1}{100}$ of a number is 5. What is the number?

1 whole = ☐ hundredths. ☐ × 5 = ☐.

$\frac{1}{20}$ of a number is 6. What is the number?

1 whole = ☐ twentieths. ☐ × 6 = ☐.

$\frac{1}{100}$ of a number is 7. The number is ☐.

If $\frac{2}{3}$ of a number is 12, what is the number?

$\frac{2}{3}$ of the number is 12.

$\frac{1}{3}$ of the number is 12 ÷ 2, or 6.

$\frac{3}{3}$ of the number is 3 × 6, or ☐.

If $\frac{3}{4}$ of a number is 15, what is the number?

$\frac{3}{4}$ of the number is 15.

$\frac{1}{4}$ of the number is 15 ÷ ☐, or ☐.

$\frac{4}{4}$ of the number is ☐ × ☐ = ☐.

If $\frac{2}{5}$ of a number is 8, what is the number?

$\frac{2}{5}$ of the number is 8.

$\frac{1}{5}$ of the number is 8 ÷ ☐, or ☐.

$\frac{5}{5}$ of the number is ☐ × ☐ = ☐.

If $\frac{3}{100}$ of a number is 6, what is the number?

$\frac{3}{100}$ of the number is 6.

$\frac{1}{100}$ of the number is 6 ÷ ☐, or ☐.

$\frac{100}{100}$ of the number is ☐ × ☐ = ☐.

If $\frac{5}{100}$ of a number is 15, what is the number?

$\frac{1}{100}$ of the number is ☐.

$\frac{100}{100}$ of the number is ☐.

The number is ☐.

If $\frac{6}{100}$ of a number is 24, what is the number?

$\frac{1}{100}$ of the number is ☐.

$\frac{100}{100}$ of the number is ☐.

The number is ☐.

Addition and Subtraction of Decimals

Add 2.56 + 4.29.

$$\begin{array}{r} {\scriptstyle 1} \\ 2.56 \\ +\ 4.29 \\ \hline \square.\square 5 \end{array}$$

If necessary, exchange 10 hundredths for 1 tenth.

Subtract 5.26 from 19.81.

$$\begin{array}{r} 1\ 9.{\scriptstyle 7}\,8\,{\scriptstyle 11}\,1 \\ -\ 5.2\ 6 \\ \hline \square\square.\square\square \end{array}$$

If necessary, exchange 1 tenth for 10 hundredths.

Add.

$$\begin{array}{r} 8.24 \\ +\ 9.16 \\ \hline \square \end{array} \qquad \begin{array}{r} 16.07 \\ +\ 2.35 \\ \hline \square \end{array} \qquad \begin{array}{r} 15.75 \\ +\ 2.41 \\ \hline \square \end{array}$$

Subtract.

$$\begin{array}{r} 56.78 \\ -\ 12.25 \\ \hline \square \end{array} \qquad \begin{array}{r} 49.06 \\ -\ 36.12 \\ \hline \square \end{array} \qquad \begin{array}{r} 25.00 \\ -\ 3.75 \\ \hline \square \end{array}$$

Add 3.672 + 1.219.

$$\begin{array}{r} {\scriptstyle 1} \\ 3.672 \\ +\ 1.219 \\ \hline \square.\square\square 1 \end{array}$$

If necessary, exchange 10 thousandths for 1 hundredth.

Subtract 4.328 from 9.582.

$$\begin{array}{r} {\scriptstyle 7}\ {\scriptstyle 12} \\ 9.5\,8\,2 \\ -\ 4.328 \\ \hline \square\square.\square\square\square \end{array}$$

If necessary, exchange 1 hundredth for 10 thousandths.

Add.

$$\begin{array}{r} 15.238 \\ +\ 12.389 \\ \hline \square \end{array}$$

Subtract.

$$\begin{array}{r} 48.621 \\ -\ 13.215 \\ \hline \square \end{array}$$

Add 6 and 5.2.

Since the second addend has tenths, write the first addend with tenths, too. 6 = 6 and no tenths = 6.0. Use 0 as a place holder in the tenths place.

$$\begin{array}{r} 6.0 \\ +\ 5.2 \\ \hline \square\square.\square \end{array}$$

Add 12, 15.3, and 4.62.

Since one of the addends has hundredths, write each addend with tenths and hundredths. Use 0 as a place holder.

$$\begin{array}{r} 12.00 \\ 15.30 \\ +\ 4.62 \\ \hline \square \end{array}$$

Add 12 and 4.9.

$$\begin{array}{r} \square\square.\square \\ +\ \square.\square \\ \hline \square \end{array}$$

Add 8.6 and 24.

$$\begin{array}{r} \square.\square \\ +\ \square\square.\square \\ \hline \square \end{array}$$

Add 4.7 and 39.

Add 2.5 and 4.96.

$$\begin{array}{r} \square.\square\square \\ +\ \square.\square\square \\ \hline \square \end{array}$$

Add 8.7 and 4.12.

Subtract 3.5 from 9.

Use 0 as a place holder in the tenths place.

$$\begin{array}{r} 9.0 \\ -\ 3.5 \\ \hline \square \end{array}$$

Subtract 2.45 from 8.

Use 0 as a place holder in the tenths place and the hundredths place.

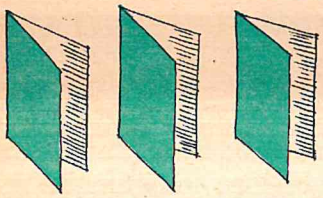
$$\begin{array}{r} 8.00 \\ -\ 2.45 \\ \hline \square \end{array}$$

Subtract 7.2 from 10.

Subtract 4 from 8.5.

Subtract 1.75 from 2.

A Whole Number Times a Decimal



A paper-folding machine folds one sheet in .2 of a second. How long does it take to fold 3 sheets?

Here are three ways of thinking out the answer:

$$\begin{array}{r} .2 \\ .2 \\ +.2 \\ \hline .6 \end{array}$$

$3 \times$ (2 tenths) $=$ 6 tenths $= .6$.

$3 \times \frac{2}{10} = \frac{6}{10} = .6$.

Answer: .6 sec.

Find $3 \times .5$.

$$\begin{array}{r} .5 \\ .5 \\ +.5 \\ \hline \square.\square \end{array}$$

$3 \times$ (5 tenths) $=$ ☐ tenths $=$ ☐.☐.

$3 \times \frac{5}{10} = \frac{\square}{10} = \square\frac{\square}{10} = \square.\square$.

$4 \times .6 = 4 \times \frac{6}{10} = \frac{\square}{10} = \square\frac{\square}{10} = \square.\square$.

$3 \times .7 = 3 \times \frac{7}{10} = \frac{\square}{10} = \square\frac{\square}{10} = \square.\square$.

$5 \times .5 = 5 \times \frac{5}{10} = \frac{\square}{10} = \square\frac{\square}{10} = \square.\square$.

When a number of tenths is multiplied by a whole number, the answer is a number of tenths. To get the answer quickly, first multiply as if there were no decimal point in the multiplicand. Then put a decimal point into the answer so that there is one digit to the right of it.

$$\begin{array}{r} .4 \\ \times\ 8 \\ \hline \end{array}$$

$8 \times 4 = \square\square$.

Put the decimal point here.

$$\begin{array}{r} .7 \\ \times\ 6 \\ \hline \end{array}$$

$6 \times 7 = \square\square$.

Put the decimal point here.

$\begin{array}{r} .2 \\ \times\ 8 \\ \hline \square.\square \end{array}$	$\begin{array}{r} .4 \\ \times\ 9 \\ \hline \square.\square \end{array}$	$\begin{array}{r} .6 \\ \times\ 5 \\ \hline \square.\square \end{array}$	$\begin{array}{r} .4 \\ \times\ 6 \\ \hline \square.\square \end{array}$	$\begin{array}{r} .9 \\ \times\ 9 \\ \hline \square.\square \end{array}$
$\begin{array}{r} .8 \\ \times\ 2 \\ \hline \end{array}$	$\begin{array}{r} .7 \\ \times\ 7 \\ \hline \end{array}$	$\begin{array}{r} .3 \\ \times\ 4 \\ \hline \end{array}$	$\begin{array}{r} .5 \\ \times\ 4 \\ \hline \end{array}$	$\begin{array}{r} .4 \\ \times\ 7 \\ \hline \end{array}$

Multiply 3×4.6.

Estimate: First round off 4.6 to the nearest unit. 3×4.6 is about $3 \times 5 = 15$.

$3 \times .6 =$
$3 \times 4 =$

$$\begin{array}{r} 4.6 \\ \times\ 3 \\ \hline 1.8 \\ 12.0 \\ \hline 13.8 \end{array}$$

Short Method

$$\begin{array}{r} 4.6 \\ \times\ 3 \\ \hline 13.8 \end{array}$$

2 place number

The estimate shows that the answer will be a 2 place number. Multiply 3×46 to get 138. Then put the decimal point here.

There is one place after the decimal point to show tenths.

Multiply 4×12.8.

Estimate: First round off 12.8 to the nearest ten. 4×12.8 is about $4 \times 10 = 40$

Short Method

$$\begin{array}{r} 12.8 \\ \times\ 4 \\ \hline \square\square.\square \end{array}$$

2 place number

decimal point

Multiply 3×486.5.

Estimate: 3×486.5 is about 3×500

$=$

Short Method

$$\begin{array}{r} 485.5 \\ \times\ 3 \\ \hline \square\square\square\square\square \end{array}$$

☐ place number

Put the decimal point ☐ places from the left.

Multiply 6×82.4.

Estimate: 6×82.4 is about $6 \times$ ☐☐☐

$=$

Short Method

$$\begin{array}{r} 82.4 \\ \times\ 6 \\ \hline \square\square\square\square \end{array}$$

☐ place number

Put the decimal point ☐ places from the left.

Multiply 8×312.9.

Estimate: 8×312.9 is about $8 \times$ ☐☐☐☐

$=$

Short Method

$$\begin{array}{r} 312.9 \\ \times\ 8 \\ \hline \square\square\square\square\square \end{array}$$

☐ place number

Put the decimal point ☐ places from the left.

Multiply 3×345.7.

Estimate: 3×345.7 is more than 3×300 or 900. 900 is a 3 place number. But it is close to 1000. The answer might be a 3 or 4 place number.

Short Method

$$\begin{array}{r} 345.7 \\ \times\ \ 3 \\ \hline 1037.1 \end{array}$$

Put the decimal point here to get a product that is about 900 or 1000.

If we make the answer a 3 place number, we get 103.71. If we make the answer a 4 place number, we get 1037.1

1037.1 is closer to 900 than 103.71 is.

Rule for placing the decimal point when you multiply a decimal fraction or a mixed decimal by a whole number:

If the multiplicand has 1 decimal place, the product has 1 decimal place.

In the exercises below, use the rule to place the decimal point in the product. Then use the estimate to see if the answer makes sense.

Estimate:

about $4 \times$ ☐

$=$ ☐.

$$\begin{array}{r} 416.2 \\ \times\ \ 4 \\ \hline \square \end{array}$$

Estimate:

about $8 \times$ ☐

$=$ ☐.

$$\begin{array}{r} 86.3 \\ \times\ \ 8 \\ \hline \square \end{array}$$

Estimate:

about $6 \times$ ☐

$=$ ☐.

$$\begin{array}{r} 75.5 \\ \times\ \ 6 \\ \hline \square \end{array}$$

Estimate:

about $7 \times$ ☐

$=$ ☐.

$$\begin{array}{r} 274.8 \\ \times\ \ 7 \\ \hline \square \end{array}$$

Estimate:

about $5 \times$ ☐

$=$ ☐.

$$\begin{array}{r} 89.9 \\ \times\ \ 5 \\ \hline \square \end{array}$$

A Decimal Times a Whole Number

$.6 \times 4 = 4 \times .6$, by the commutative law of multiplication.

$$\begin{array}{r} .6 \\ \times\ 4 \\ \hline 2.4 \end{array}$$

$4 \times 6 = 24.$ Put decimal point here.

or

$$\begin{array}{r} 4 \\ \times\ .6 \\ \hline 2.4 \end{array}$$

$6 \times 4 = 24.$ Put decimal point here.

$.8 \times 7 = 7 \times .8$.

$$\begin{array}{r} .8 \\ \times\ 7 \\ \hline \square\square \end{array}$$

$7 \times 8 = \square\square.$ Put decimal point here.

or

$$\begin{array}{r} 7 \\ \times\ .8 \\ \hline \square\square \end{array}$$

$8 \times 7 = \square.$ Put decimal point here.

Multiply.

$$\begin{array}{r} 12 \\ \times .4 \\ \hline \square \end{array} \qquad \begin{array}{r} 9 \\ \times .3 \\ \hline \square \end{array} \qquad \begin{array}{r} 15 \\ \times .4 \\ \hline \square \end{array} \qquad \begin{array}{r} 25 \\ \times .5 \\ \hline \square \end{array}$$

Each product has ☐ decimal place.

Multiply 4.2×7.

Estimate: about $4 \times 7 = 28$.

$$\begin{array}{r} 7 \\ \times 4.2 \\ \hline \square.\square \\ \square\square.\square \\ \hline \square\square.\square \end{array}$$

$.2 \times 7 =$

$4 \times 7 =$

1 decimal place

1 decimal place

Check.

$$\begin{array}{r} 4.2 \\ \times\ 7 \\ \hline \square\square.\square \end{array}$$

Multiply 32.1×8.

Estimate: about $30 \times 8 =$ ☐.

$$\begin{array}{r} 8 \\ \times 32.1 \\ \hline .\square \\ \square\square.\square \\ \square\square\square.\square \\ \hline \square\square\square.\square \end{array}$$

$.1 \times 8 =$

$2 \times 8 =$

$30 \times 8 =$

1 decimal place

1 decimal place

Check.

$$\begin{array}{r} 32.1 \\ \times\ 8 \\ \hline \square \end{array}$$

Multiply 26.5 by 9.

Estimate: about ☐ $\times 9 =$ ☐.

$$\begin{array}{r} 9 \\ \times 26.5 \\ \hline \end{array}$$

$.5 \times 9 =$

$6 \times 9 =$

$20 \times 9 =$

$$\square$$

Check.

$$\begin{array}{r} 26.5 \\ \times\ 9 \\ \hline \square \end{array}$$

Short Division

There were 536 children in a school. They were divided into 4 equal groups for lunch. How many were in each group?

Here is a new short way of doing the division. Think of 536 as 5 hundreds + 3 tens + 6 units. Do the division in 3 steps.

Step I Without breaking up any hundreds, divide 5 hundreds into 4 equal parts. Each part has 1 hundred. Write this 1 in the hundreds place as shown at the right. There is 1 hundred left.

$$4\overline{)5\ 3\ 6}\quad 1$$

1 hundred = 10 tens.
10 tens + 3 tens = 13 tens.

Step II Exchange the 1 hundred left for 10 tens and combine with the 3 tens to get 13 tens. Without breaking up any tens, divide 13 tens into 4 equal parts. Each part has 3 tens. Write the 3 in the tens place, as shown at the right. There is 1 ten left.

$$4\overline{)5\ 3\ 6}\quad 1\ 3$$

1 ten = 10 units.
10 units + 6 units = 16 units.

Step III Exchange the 1 ten for 10 units, and combine with the 6 units to get 16 units. Divide 16 units into 4 equal parts. Each part has 4 units. Write the 4 in the units place, as shown at the right.

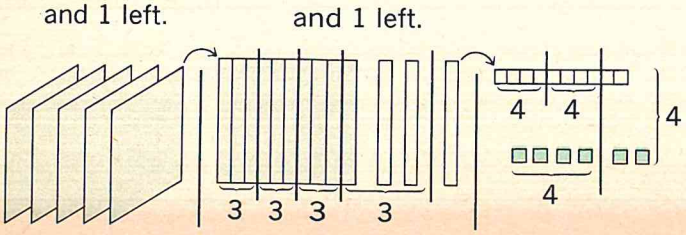
$$4\overline{)5\ 3\ 6}\quad 1\ 3\ 4$$

The answer is: 1 hundred + 3 tens + 4 units = 134.

Hundreds
$5 \div 4 = 1$
and 1 left.

Tens
$13 \div 4 = 3$
and 1 left.

Units
$16 \div 4 = 4$.



Divide by short division.

$3\overline{)4\ 7\ 1}$ $5\overline{)6\ 1\ 5}$ $2\overline{)7\ 6}$

$6\overline{)7\ 9\ 2}$ $2\overline{)8\ 5\ 2}$ $4\overline{)4\ 9\ 2}$

How many 5's are there in 675? This question can be answered by short division, too. Think of 675 as 6 hundreds + 7 tens + 5 units.

Step I How many hundreds of 5's are there in 6 hundred?

Answer: 1 hundred, and 1 hundred left.

$$5\overline{)6\ 7\ 5}\quad 1$$

Step II 1 hundred left + 7 tens = 17 tens. How many tens of 5's are there in 17 tens?

Answer: 3 tens, and 2 tens left.

$$5\overline{)6\ 7\ 5}\quad 1\ 3$$

Step III 2 tens + 5 units = 25. How many 5's are there in 25?

Answer: 5

$$5\overline{)6\ 7\ 5}\quad 1\ 3\ 5$$

Final answer: 1 hundred + 3 tens + 5 = 135.

How many 4's are there in 128?

Step I How many hundreds of 4's are there in 1 hundred? There are none. Exchange 1 hundred for 10 tens and go on to Step II: How many tens of 4's are there in 12 tens?

$$4\overline{)1\ 2\ 8}\quad \square\square$$

Divide by short division.

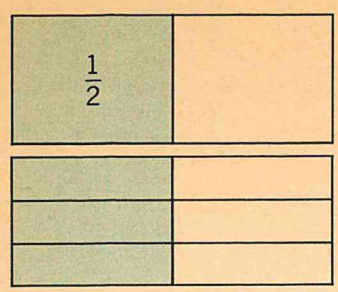
$2\overline{)1\ 1\ 2}$ $3\overline{)6\ 0\ 9}$ $4\overline{)6\ 0}$

$5\overline{)4\ 0\ 5}$ $9\overline{)1\ 9\ 8}$ $8\overline{)2\ 5\ 6}$

$7\overline{)3\ 7\ 8}$ $6\overline{)3\ 5\ 4}$ $5\overline{)1\ 2\ 5}$

A Unit Fraction Times a Unit Fraction

3 children have $\frac{1}{2}$ a small cake to share. Each one takes $\frac{1}{3}$ of the half-cake. What part of a cake does each one get?



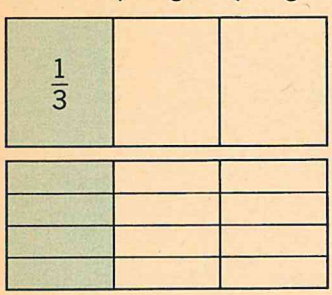
Here is 1 whole cake. The colored part is $\frac{1}{2}$ of a cake.

Divide each half into thirds, showing 3 equal parts in each half. Then there are 6 equal parts in the whole.

Each part is $\frac{1}{6}$. Each child gets $\frac{1}{6}$ of the cake.

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.

Find $\frac{1}{4} \times \frac{1}{3}$ or $\frac{1}{4}$ of $\frac{1}{3}$.

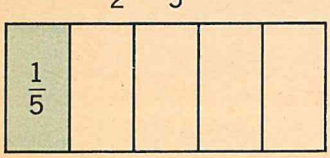


The colored part is $\frac{1}{3}$ of the whole.

Divide each third into fourths, showing 4 equal parts in each third. There are ☐ equal parts in the whole.

$\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{\square}$. $\frac{1}{4} \times \frac{1}{3} = \frac{1}{\square}$.

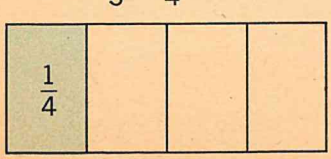
Find $\frac{1}{2} \times \frac{1}{5}$.



Divide each fifth into 2 equal parts. Then there are ☐ equal parts in the whole.

$\frac{1}{2}$ of $\frac{1}{5} = \frac{1}{\square}$. $\frac{1}{2} \times \frac{1}{5} = \frac{1}{\square}$.

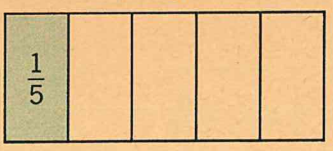
Find $\frac{1}{3}$ of $\frac{1}{4}$.



Divide each fourth into ☐ equal parts. Then there are ☐ equal parts in the whole.

$\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{\square}$. $\frac{1}{3} \times \frac{1}{4} = \frac{1}{\square}$.

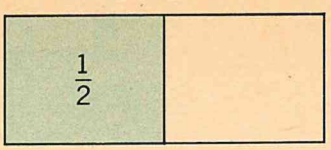
Find $\frac{1}{4}$ of $\frac{1}{5}$.



Divide each fifth into ☐ equal parts. Then there are ☐ equal parts in the whole.

$\frac{1}{4}$ of $\frac{1}{5} = \frac{1}{\square}$. $\frac{1}{4} \times \frac{1}{5} = \frac{1}{\square}$.

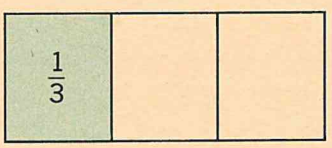
Find $\frac{1}{2}$ of $\frac{1}{2}$.



Divide each half into ☐ equal parts. Then there are ☐ equal parts in the whole.

$\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{\square}$. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{\square}$.

Find $\frac{1}{3}$ of $\frac{1}{3}$.



Divide each third into ☐ equal parts. Then there are ☐ equal parts in the whole.

$\frac{1}{3}$ of $\frac{1}{3} = \frac{1}{\square}$. $\frac{1}{3} \times \frac{1}{3} = \frac{1}{\square}$.

The exercises in the first column have these answers:

$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$. $\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$. $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$.

There is a quick way of getting these answers.

To multiply unit fractions, multiply the numerators to get the numerator of the product. Multiply the denominators to get the denominator of the product.

Multiply in the quick way.

$\frac{1}{4} \times \frac{1}{7} = \frac{1 \times 1}{4 \times 7} = \frac{\square}{\square}$. $\frac{1}{3} \times \frac{1}{5} = \frac{1 \times 1}{3 \times 5} = \frac{\square}{\square}$.

$\frac{1}{6} \times \frac{1}{9} = \frac{1 \times 1}{6 \times 9} = \frac{\square}{\square}$. $\frac{1}{10} \times \frac{1}{10} = \frac{1 \times 1}{10 \times 10} = \frac{\square}{\square}$.

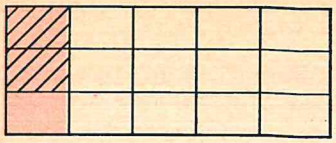
$\frac{1}{4} \times \frac{1}{4} = \frac{1 \times 1}{4 \times 4} = \frac{\square}{\square}$. $\frac{1}{2} \times \frac{1}{5} = \frac{1 \times 1}{2 \times 5} = \frac{\square}{\square}$.

$\frac{1}{4} \times \frac{1}{5} = \frac{\square}{\square}$. $\frac{1}{6} \times \frac{1}{4} = \frac{\square}{\square}$.

$\frac{1}{2} \times \frac{1}{10} = \frac{\square}{\square}$. $\frac{1}{3} \times \frac{1}{10} = \frac{\square}{\square}$.

Any Fraction Times a Unit Fraction

Find $\frac{2}{3} \times \frac{1}{5}$.



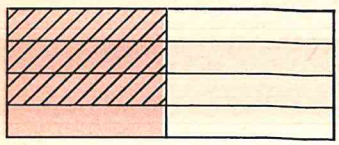
$\frac{2}{3} = 2 \times \frac{1}{3}$.

$\frac{2}{3} \times \frac{1}{5} = \left(2 \times \frac{1}{3}\right) \times \frac{1}{5}$.

Use the associative law of multiplication.

$\left(2 \times \frac{1}{3}\right) \times \frac{1}{5} = 2 \times \left(\frac{1}{3} \times \frac{1}{5}\right) = 2 \times \frac{1}{15} = \frac{2}{15}$.

Find $\frac{3}{4} \times \frac{1}{2}$.



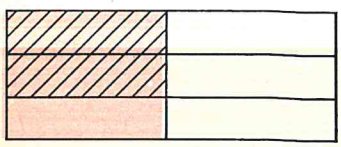
$\frac{3}{4} = 3 \times \frac{1}{4}$.

$\frac{3}{4} \times \frac{1}{2} = \left(3 \times \frac{1}{4}\right) \times \frac{1}{2}$.

Use the associative law of multiplication.

$\left(3 \times \frac{1}{4}\right) \times \frac{1}{2} = 3 \times \left(\frac{1}{4} \times \frac{1}{2}\right) = 3 \times \frac{1}{\square} = \frac{\square}{\square}$.

Find $\frac{2}{3} \times \frac{1}{2}$.



$\frac{2}{3} = 2 \times \frac{1}{3}$.

$\frac{2}{3} \times \frac{1}{2} = \left(2 \times \frac{1}{3}\right) \times \frac{1}{2}$.

Use the associative law of multiplication.

$\left(2 \times \frac{1}{3}\right) \times \frac{1}{2} = 2 \times \left(\frac{1}{3} \times \frac{1}{2}\right) = 2 \times \frac{1}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$.

Multiply with the help of the associative law of multiplication.

$\frac{3}{4} \times \frac{1}{5} = \left(\square \times \frac{1}{4}\right) \times \frac{1}{5} = \square \times \left(\frac{1}{4} \times \frac{1}{5}\right)$

$= \square \times \frac{1}{\square} = \frac{\square}{\square}$.

$\frac{2}{5} \times \frac{1}{7} = \left(\square \times \frac{1}{5}\right) \times \frac{1}{7} = \square \times \left(\frac{1}{5} \times \frac{1}{7}\right)$

$= \square \times \frac{1}{\square} = \frac{\square}{\square}$.

$\frac{3}{10} \times \frac{1}{2} = \left(\square \times \frac{1}{10}\right) \times \frac{1}{2} = \square \times \left(\frac{1}{10} \times \frac{1}{2}\right)$

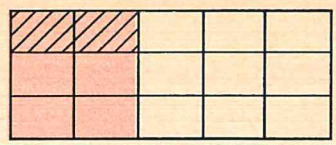
$= \square \times \frac{1}{\square} = \frac{\square}{\square}$.

$\frac{7}{5} \times \frac{1}{2} = \left(\square \times \frac{1}{5}\right) \times \frac{1}{2} = \square \times \left(\frac{1}{5} \times \frac{1}{2}\right)$

$= \square \times \frac{1}{\square} = \frac{\square}{\square}$.

A Unit Fraction Times Any Fraction

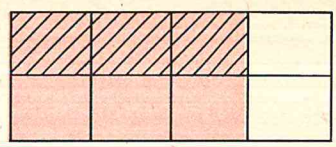
Find $\frac{1}{3} \times \frac{2}{5}$.



$\frac{1}{3} \times \frac{1}{5} = \frac{1}{\square}$.

$\frac{1}{3} \times \frac{2}{5} = \frac{\square}{\square}$.

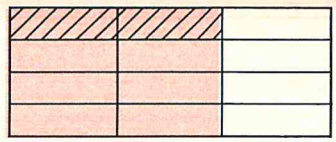
Find $\frac{1}{2} \times \frac{3}{4}$.



$\frac{1}{2} \times \frac{1}{4} = \frac{1}{\square}$.

$\frac{1}{2} \times \frac{3}{4} = \frac{\square}{\square}$.

Find $\frac{1}{4} \times \frac{2}{3}$.



$\frac{1}{4} \times \frac{1}{3} = \frac{1}{\square}$.

$\frac{1}{4} \times \frac{2}{3} = \frac{\square}{\square}$.

Multiply with the help of the associative law of multiplication.

$\frac{1}{5} \times \frac{3}{4} = \frac{1}{5} \times \left(\frac{1}{4} \times 3\right) = \left(\frac{1}{5} \times \frac{1}{4}\right) \times 3$

$= \frac{1}{\square} \times 3 = \frac{\square}{\square}$.

$\frac{1}{6} \times \frac{5}{2} = \frac{1}{6} \times \left(\frac{1}{2} \times \square\right) = \left(\frac{1}{6} \times \frac{1}{2}\right) \times \square$

$= \frac{1}{\square} \times \square = \frac{\square}{\square}$.

Quick way of multiplying any fraction times a unit fraction, or a unit fraction times any fraction:

Multiply the numerators to get the numerator of the product. Multiply the denominators to get the denominator of the product.

$\frac{1}{5} \times \frac{2}{9} = \frac{1 \times 2}{5 \times 9} = \frac{\square}{\square}$.

$\frac{3}{5} \times \frac{1}{2} = \frac{3 \times 1}{5 \times 2} = \frac{\square}{\square}$.

$\frac{1}{4} \times \frac{5}{6} = \frac{\square}{\square}$.

$\frac{1}{8} \times \frac{2}{3} = \frac{\square}{\square} = \frac{\square}{\square}$.

$\frac{3}{10} \times \frac{1}{3} = \frac{\square}{\square} = \frac{\square}{\square}$.

$\frac{3}{5} \times \frac{1}{4} = \frac{\square}{\square}$.

$\frac{10}{7} \times \frac{1}{2} = \frac{\square}{\square} = \frac{\square}{\square}$.

$\frac{5}{3} \times \frac{1}{2} = \frac{\square}{\square}$.

$\frac{1}{3} \times \frac{10}{9} = \frac{\square}{\square}$.

$\frac{1}{2} \times \frac{5}{4} = \frac{\square}{\square}$.

Any Fraction Times Any Fraction

You already know how to multiply a whole number times a fraction. (See page 59.)

Example $2 \times \frac{4}{15} = \frac{2 \times 4}{15} = \frac{8}{15}$.

You already know how to multiply a unit fraction times any fraction. (See page 74.)

Example $\frac{1}{3} \times \frac{4}{5} = \frac{1 \times 4}{3 \times 5} = \frac{4}{15}$.

Using this knowledge and the associative law of multiplication, you can multiply any two fractions.

Example $\frac{2}{3} \times \frac{4}{5} = \left(2 \times \frac{1}{3}\right) \times \frac{4}{5}$

$= 2 \times \left(\frac{1}{3} \times \frac{4}{5}\right) = 2 \times \frac{4}{15} = \frac{8}{15}$.

Multiply with the help of the associative law of multiplication.

$\frac{3}{5} \times \frac{4}{5} = \left(3 \times \frac{1}{5}\right) \times \frac{4}{5} = 3 \times \left(\frac{1}{5} \times \frac{4}{5}\right)$

$= 3 \times \frac{\square}{\square} = \frac{\square}{\square}$.

$\frac{2}{5} \times \frac{3}{2} = \left(2 \times \frac{1}{5}\right) \times \frac{3}{2} = \square \times \left(\frac{1}{5} \times \frac{3}{2}\right)$

$= \square \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$.

$\frac{3}{8} \times \frac{2}{5} = \left(\square \times \frac{1}{8}\right) \times \frac{2}{5} = \square \times \left(\frac{1}{8} \times \frac{2}{5}\right)$

$= \square \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$.

Quick way to multiply any two fractions:

Multiply the numerators to get the numerator of the product. Multiply the denominators to get the denominator of the product.

$\frac{2}{7} \times \frac{5}{2} =$	$\frac{2}{3} \times \frac{3}{5} =$
$\frac{3}{8} \times \frac{2}{3} =$	$\frac{4}{9} \times \frac{3}{2} =$
$\frac{5}{8} \times \frac{1}{2} =$	$\frac{1}{4} \times \frac{1}{3} =$
$\frac{3}{2} \times \frac{4}{3} =$	$\frac{3}{4} \times \frac{4}{3} =$

Writing a Whole Number as a Fraction

$\frac{4}{1} = 4 \div 1 = 4$. 4 equals the fraction $\frac{4}{1}$.

$\frac{5}{1} = 5 \div 1 = 5$. 5 equals the fraction $\frac{5}{1}$.

To change a whole number to a fraction, use the whole number as numerator, and use 1 as the denominator.

Change each whole number to a fraction.

$10 =$	$8 =$	$15 =$
$6 =$	$3 =$	$2 =$

If a multiplier or multiplicand is a whole number, you may change it to a fraction.

Example $2 \times \frac{3}{5} = \frac{2}{1} \times \frac{3}{5} = \frac{2 \times 3}{1 \times 5} = \frac{\square}{\square}$.

Change the whole number to a fraction. Then multiply.

$5 \times \frac{2}{3} = \frac{\square}{\square} \times \frac{2}{3} = \frac{\square}{\square}$.	$\frac{3}{8} \times 4 = \frac{3}{8} \times \frac{\square}{\square} = \frac{\square}{\square}$.
$6 \times \frac{1}{2} = \frac{\square}{\square} \times \frac{1}{2} = \frac{\square}{\square}$.	$\frac{3}{5} \times 2 = \frac{3}{5} \times \frac{\square}{\square} = \frac{\square}{\square}$.

If a multiplier or multiplicand is a mixed number, you may change it to an improper fraction.

Example $4\frac{1}{2} \times 2\frac{1}{3} = \frac{9}{2} \times \frac{7}{3} = \frac{\square}{\square}$.

Change each whole or mixed number to an improper fraction. Then multiply.

$2\frac{1}{2} \times 1\frac{1}{2} = \frac{\square}{2} \times \frac{\square}{2} = \frac{\square}{\square}$.

$6 \times 1\frac{1}{3} = \frac{\square}{\square} \times \frac{\square}{3} = \frac{\square}{3} = \square$.

$2\frac{1}{5} \times 4 =$

$3\frac{1}{2} \times 3 =$

A New Look at Changing Fractions to Higher or Lower Terms

A number is not changed when it is multiplied by 1. (See page 54.)

Example $\frac{1}{2} \times 1 = \frac{1}{2}$.

There are many ways of writing 1 as a fraction. (See page 54.)

Example $1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5}$, and so on.

Combining these two facts, you get a way of changing a fraction without changing its value.

For example, these facts can be used to change to higher terms:

$$\frac{1}{2} = \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{2}{2} = \frac{2}{4}, \text{ or}$$

$$\frac{1}{2} = \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{3}{3} = \frac{3}{6}.$$

To raise a fraction to higher terms, multiply the fraction by a fraction that is equal to 1.

Change $\frac{1}{3}$ to sixths. $\frac{1}{3} \times \frac{\square}{\square} = \frac{\square}{6}$.

Change $\frac{1}{2}$ to eighths. $\frac{1}{2} \times \frac{\square}{\square} = \frac{\square}{8}$.

Change $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to twelfths. $\frac{1}{2} \times \frac{\square}{\square} = \frac{\square}{12}$. $\frac{1}{3} \times \frac{\square}{\square} = \frac{\square}{12}$. $\frac{1}{4} \times \frac{\square}{\square} = \frac{\square}{12}$.

$\frac{4}{1} \times \frac{3}{3} = \frac{4 \times 3}{1 \times 3}$. This statement can be read backwards: $\frac{4 \times 3}{1 \times 3} = \frac{4}{1} \times \frac{3}{3}$. You can use this fact to reduce a fraction to lower terms:

$$\frac{12}{3} = \frac{4 \times 3}{1 \times 3} = \frac{4}{1} \times \frac{3}{3} = 4 \times 1 = 4.$$

$$\frac{10}{15} = \frac{2 \times 5}{3 \times 5} = \frac{2}{3} \times \frac{5}{5} = \frac{2}{3} \times 1 = \frac{2}{3}.$$

To reduce a fraction to lower terms, first find a number that is a factor of both numerator and denominator.

Example 5 is a factor of 10 and 15.

Using this factor, write the numerator and denominator as a product of factors.

Example $10 = 2 \times 5$, and $15 = 3 \times 5$.

Then write the fraction as a product of two fractions, one of which equals 1.

Example $\frac{2 \times 5}{3 \times 5} = \frac{2}{3} \times \frac{5}{5} = \frac{2}{3} \times 1 = \frac{2}{3}$.

Reduce to lower terms.

$\frac{8}{20} = \frac{2 \times \square}{5 \times \square} = \frac{2}{5} \times \frac{\square}{\square} = \frac{2}{5} \times 1 = \frac{2}{5}$.

$\frac{6}{9} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} \times 1 = \frac{\square}{\square}$.

$\frac{8}{10} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} \times 1 = \frac{\square}{\square}$.

$\frac{15}{25} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} \times 1 = \frac{\square}{\square}$.

$\frac{12}{15} =$

$\frac{6}{8} =$

The Reciprocal of a Number

$\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$. $\frac{3}{2} \times \frac{2}{3} = \frac{6}{6} = 1$.

The reciprocal of a number is the number you must multiply it by to get 1 as the product.

$\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$. $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$.

To get the reciprocal of a number, interchange the numerator and denominator.

The reciprocal of $\frac{2}{5}$ is $\frac{\square}{\square}$. Check: $\frac{2}{5} \times \frac{\square}{\square} = \frac{\square}{\square} = 1$.

The reciprocal of $\frac{3}{1}$ is $\frac{\square}{\square}$. Check: $\frac{3}{1} \times \frac{\square}{\square} = \frac{\square}{\square} = 1$.

The reciprocal of $\frac{1}{4}$ is $\frac{\square}{\square}$. Check: $\frac{1}{4} \times \frac{\square}{\square} = \frac{\square}{\square} = 1$.

A Second Look at the Distributive Law

If a, b, and c stand for numbers, the distributive law says that $a \times (b + c) = (a \times b) + (a \times c)$.

This statement can also be read backwards:

$$(a \times b) + (a \times c) = a \times (b + c).$$

Example $(2 \times 3) + (2 \times 5) = 2 \times (3 + 5)$.

The 2 appears twice here.

The 2 appears once here.

Example $(5 \times 7) + (5 \times 4) = 5 \times (7 + 4)$.

The 5 appears twice here.

The 5 appears once here.

Use the distributive law backwards to complete these statements:

$(4 \times 3) + (4 \times 2) = \square \times (3 + 2)$.

$(6 \times 1) + (6 \times 5) = \square \times (1 + 5)$.

$(\frac{1}{2} \times 3) + (\frac{1}{2} \times 5) = \square \times (3 + 5)$.

$(3 \times 2) + (3 \times 4) = 3 \times (\square + \square)$.

$(7 \times 1) + (7 \times 2) = 7 \times (\square + \square)$.

$(\frac{1}{3} \times 2) + (\frac{1}{3} \times 1) = \frac{1}{3} \times (\square + \square)$.

A New Look at Addition of Like Fractions

We can use the distributive law and the commutative law of multiplication to help us add like fractions.

Example Add $\frac{2}{5} + \frac{1}{5}$.

$\frac{2}{5}$ means $2 \times \frac{1}{5}$. $\frac{1}{5}$ means $1 \times \frac{1}{5}$.

But $2 \times \frac{1}{5} = \frac{1}{5} \times 2$. $1 \times \frac{1}{5} = \frac{1}{5} \times 1$.

$$\frac{2}{5} + \frac{1}{5} = (2 \times \tfrac{1}{5}) + (1 \times \tfrac{1}{5})$$

$$= (\tfrac{1}{5} \times 2) + (\tfrac{1}{5} \times 1)$$

$$= \tfrac{1}{5} \times (2 + 1) = \tfrac{1}{5} \times (3) = \tfrac{3}{5}.$$

The commutative law of multiplication was used in this step.

The distributive law was used in this step.

Add, with the help of the commutative law of multiplication and the distributive law.

$$\frac{3}{8} + \frac{2}{8} = \left(\square \times \tfrac{1}{8}\right) + \left(\square \times \tfrac{1}{8}\right)$$

$$= \left(\tfrac{1}{8} \times \square\right) + \left(\tfrac{1}{8} \times \square\right)$$

$$= \tfrac{1}{8} \times \left(\square + \square\right) = \tfrac{1}{8} \times \square = \frac{\square}{8}.$$

$$\frac{3}{10} + \frac{4}{10} = \left(\square \times \tfrac{1}{10}\right) + \left(\square \times \tfrac{1}{10}\right)$$

$$= \left(\tfrac{1}{10} \times \square\right) + \left(\tfrac{1}{10} \times \square\right)$$

$$= \tfrac{1}{10} \times \left(\square + \square\right) = \tfrac{1}{10} \times \square = \frac{\square}{10}.$$

$$\frac{2}{7} + \frac{6}{7} = \left(\square \times \tfrac{1}{7}\right) + \left(\square \times \tfrac{1}{7}\right)$$

$$= \left(\tfrac{1}{7} \times \square\right) + \left(\tfrac{1}{7} \times \square\right)$$

$$= \tfrac{1}{7} \times \left(\square + \square\right) = \tfrac{1}{7} \times \square = \frac{\square}{7} = \square\frac{\square}{7}.$$

A New Look at Addition of Unlike Fractions

Change each set of fractions to fractions with a common denominator. Use the law of 1.

$\frac{1}{2}, \frac{1}{3}$ $\quad \frac{1}{2} \times \frac{\square}{\square} = \frac{\square}{6}; \frac{1}{3} \times \frac{\square}{\square} = \frac{\square}{6}.$

$\frac{1}{3}, \frac{1}{6}$ $\quad \frac{1}{3} \times \frac{\square}{\square} = \frac{\square}{6}; \frac{1}{6} \times \frac{1}{1} = \frac{\square}{\square}.$

$\frac{1}{3}, \frac{1}{4}$ $\quad \frac{1}{3} \times \frac{\square}{\square} = \frac{\square}{\square}; \frac{1}{4} \times \frac{\square}{\square} = \frac{\square}{\square}.$

Add with the help of the law of 1, the commutative law of multiplication, and the distributive law.

Add $\frac{1}{2} + \frac{1}{5}$.

The common denominator is 10. Change both fractions to tenths.

$\frac{1}{2} \times \frac{5}{5} = \frac{5}{10}$.

$\frac{1}{5} \times \frac{2}{2} = \frac{2}{10}$.

$$\frac{5}{10} + \frac{2}{10}$$

$$= \left(5 \times \tfrac{1}{10}\right) + \left(2 \times \tfrac{1}{10}\right)$$

$$= \left(\tfrac{1}{10} \times 5\right) + \left(\tfrac{1}{10} \times 2\right)$$

$$= \tfrac{1}{10} \times (5 + 2)$$

$$= \tfrac{1}{10} \times 7 = \frac{7}{10}.$$

Underline the steps that use the law of 1.

Circle the step that uses the commutative law of multiplication.

Draw a box around the step that uses the distributive law.

A Whole Number Times a Decimal

$3 \times .02 = 3 \times \frac{2}{100} = \frac{6}{100} = .06.$

$5 \times .04 = 5 \times \frac{\square}{100} = \frac{\square}{100} = .\square\square.$

$4 \times .35 = 4 \times \frac{\square}{100} = \frac{\square}{100} = \square\frac{\square}{100} = \square.\square\square.$

When you multiply a decimal fraction by a whole number, if the multiplicand has hundredths (2 decimal places), the product has hundredths (2 decimal places).

To get the product quickly, first multiply as if there were no decimal point in the mutiplicand. Then put a decimal point into the answer so that there are two digits to the right of it.

$$\begin{array}{r} .5\,2 \\ \times\ 6 \\ \hline \square.\square\square \end{array}$$

2 decimal places

$6 \times 52 = \square\square.$

$$\begin{array}{r} .7\,5 \\ \times\ 5 \\ \hline \square.\square\square \end{array}$$

2 decimal places

$5 \times 75 = \square\square.$

$\begin{array}{r} .0\,3 \\ \times\ 3 \\ \hline .\square\square \end{array}$	$\begin{array}{r} .1\,2 \\ \times\ 7 \\ \hline .\square\square \end{array}$	$\begin{array}{r} .6\,3 \\ \times\ 5 \\ \hline \square.\square\square \end{array}$	$\begin{array}{r} .8\,2 \\ \times\ 6 \\ \hline \square.\square\square \end{array}$
$\begin{array}{r} .0\,2 \\ \times\ 2 \\ \hline \end{array}$	$\begin{array}{r} .2\,6 \\ \times\ 3 \\ \hline \end{array}$	$\begin{array}{r} .7\,2 \\ \times\ 6 \\ \hline \end{array}$	$\begin{array}{r} .9\,1 \\ \times\ 4 \\ \hline \end{array}$

$2 \times (3.05) = 2 \times (3 + .05) = (2 \times 3) + (2 \times .05)$
$= 6 + .10 = 6.10.$

$3 \times (2.43) = 3 \times (2 + .43) = (3 \times 2) + (3 \times .43)$
$= \square + \square.\square\square = \square.\square\square.$

When you multiply a mixed decimal by a whole number, if the multiplicand has 2 decimal places, the product has 2 decimal places.

$2 \times 2.13 = \square.\square\square.$ $\quad 3 \times 2.11 = \square.\square\square.$

$4 \times 1.12 = \square.$ $\quad 2 \times 1.14 = \square.$

$\begin{array}{r} 5.67 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 8.32 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 9.07 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 2.45 \\ \times 7 \\ \hline \end{array}$

A Decimal Times a Whole Number

$.02 \times 3 = 3 \times .02$, by the commutative law of multiplication.

$$\begin{array}{r} .02 \\ \times 3 \\ \hline .06 \end{array}$$

$3 \times 2 = 6.$ Put decimal point here.

or

$$\begin{array}{r} 3 \\ \times .02 \\ \hline .06 \end{array}$$

$2 \times 3 = 6.$ Put decimal point here.

$.40 \times 2 = 2 \times .40.$

$$\begin{array}{r} .40 \\ \times 2 \\ \hline \square\square \end{array}$$

$2 \times 40 = \square\square.$ Put decimal point here.

$$\begin{array}{r} 2 \\ \times .40 \\ \hline \square\square \end{array}$$

$40 \times 2 = \square\square.$ Put decimal point here.

Multiply.

$\begin{array}{r} 5 \\ \times .06 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times .07 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times .30 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ \times .50 \\ \hline \end{array}$

Each product has $\square$ decimal places.

Multiply 5.67×8.

Estimate: about $6 \times 8 = 48$.

$$\begin{array}{r} 8 \\ \times 5.67 \\ \hline \end{array} \quad \leftarrow \text{2 decimal places}$$

$.07 \times 8 = \square.\square\square$

$.60 \times 8 = \square\square.\square\square$

$5 \times 8 = \square\square\square.\square\square$

$\square\square\square.\square\square$ ← 2 decimal places

Check.

$$\begin{array}{r} 5.67 \\ \times 8 \\ \hline \square \end{array}$$

Multiply 8.32×5.

Estimate: about $8 \times 5 = \square$.

$$\begin{array}{r} 5 \\ \times 8.32 \\ \hline \end{array} \quad \leftarrow \text{2 decimal places}$$

$.02 \times 5 = \square.\square\square$

$.30 \times 5 = \square\square.\square\square$

$8 \times 5 = \square\square\square.\square\square$

$\square\square\square.\square\square$ ← 2 decimal places

Check.

$$\begin{array}{r} 8.32 \\ \times 5 \\ \hline \square \end{array}$$

$$\begin{array}{r} 6 \\ \times 9.47 \\ \hline \end{array}$$

$.07 \times 6 =$

$.40 \times 6 =$

$9 \times 6 = \square$

Check.

$$\begin{array}{r} 9.47 \\ \times 6 \\ \hline \square \end{array}$$

Short Division with a Four-Place Dividend

$3\overline{)6\ 0\ 9\ 3}$ means

$3\overline{)6 \text{ thousands} + 0 \text{ hundreds} + 9 \text{ tens} + 3}$

$\underset{3\overline{)6 \text{ thousands}}}{2 \text{ thousands}} + \underset{3\overline{)0 \text{ hundreds}}}{0 \text{ hundreds}} + \underset{3\overline{)9 \text{ tens}}}{3 \text{ tens}} + \underset{3\overline{)3}}{1}$

Short way of writing this exercise → $3\overline{)6\ 0\ 9\ 3}$ (2 0 3 1)

Short Division with a Five-Place Dividend

$3\overline{)3\ 0\ 6\ 3\ 9}$ means

$\underset{3\overline{)3 \text{ ten thousands}}}{1 \text{ ten thousands}} + \underset{3\overline{)0 \text{ thousands}}}{0 \text{ thousands}}$

$\underset{+\ 3\overline{)6 \text{ hundreds}}}{2 \text{ hundreds}} + \underset{3\overline{)3 \text{ tens}}}{1 \text{ ten}} + \underset{3\overline{)9}}{3}$

Short way of writing this exercise → $3\overline{)3\ 0\ 6\ 3\ 9}$ (1 0 2 1 3)

$4\overline{)6\ 4\ 5\ 2}$ means $4\overline{)6 \text{ thousands} + 4 \text{ hundreds} + 5 \text{ tens} + 2}$

6 thousands = 4 thousands + 2 thousands.
4 ÷ 4 = 1. Exchange 2 thousands for 20 hundreds.
20 hundreds + 4 hundreds = 24 hundreds.
24 ÷ 4 = 6.

$4\overline{)4 \text{ thousands} + 24 \text{ hundreds} + 5 \text{ tens} + 2}$ (1 thousand + 6 hundreds)

5 tens = 4 tens + 1 ten.
4 ÷ 4 = 1. Exchange 1 ten for 10.
10 + 2 = 12. 12 ÷ 4 = 3.

$4\overline{)4 \text{ thousands} + 24 \text{ hundreds} + 4 \text{ tens} + 12}$ (1 thousand + 6 hundreds + 1 ten + 3)

Short way of writing this exercise → $4\overline{)6\ 4\ 5\ 2}$ (1 6 1 3)

$4\overline{)6\ 4\ 4\ 2\ 0}$ (1 ←) 6 ten thousands ÷ 4 = 1 ten thousand, and 2 ten thousands left.

2 ten thousands = 20 thousands.
20 thousands + 4 thousands = 24 thousands.

$4\overline{)6\ 4\ 4\ 2\ 0}$ (1 6 ←) 24 thousands ÷ 4 = 6 thousands.

$4\overline{)6\ 4\ 4\ 2\ 0}$ (1 6 1 ←) 4 hundreds ÷ 4 = 1 hundred.

$4\overline{)6\ 4\ 4\ 2\ 0}$ (1 6 1 0 ←) 2 tens ÷ 4 = 0 tens, and 2 tens left.

2 tens = 20. 20 + 0 = 20.

$4\overline{)6\ 4\ 4\ 2\ 0}$ (1 6 1 0 5 ←) 20 ÷ 4 = 5.

Divide by short division.

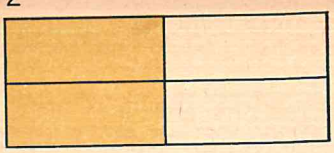
$3\overline{)6\ 1\ 3\ 5}$	$4\overline{)6\ 2\ 5\ 6}$
$7\overline{)8\ 6\ 3\ 8}$	$6\overline{)6\ 3\ 5\ 4}$
$5\overline{)2\ 4\ 4\ 5}$	$3\overline{)2\ 4\ 6\ 3}$

Divide by short division.

$2\overline{)2\ 4\ 8\ 6\ 4}$	$3\overline{)1\ 5\ 7\ 2\ 9}$
$6\overline{)2\ 1\ 6\ 0\ 0}$	$4\overline{)5\ 2\ 6\ 4\ 8}$
$4\overline{)8\ 1\ 2\ 3\ 2}$	$5\overline{)6\ 0\ 0\ 0\ 0}$

A Unit Fraction Divided by a Whole Number

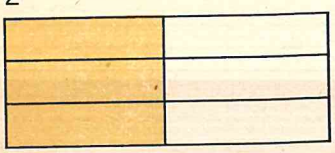
$\frac{1}{2} \div 2$



Divide each half into 2 equal parts. Then there are $\square$ equal parts in the whole. Each part is $\frac{1}{\square}$.

$\frac{1}{2} \div 2 = \frac{1}{\square}$.

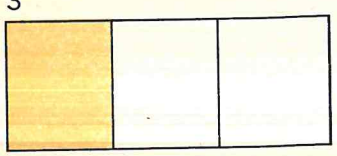
$\frac{1}{2} \div 3$



Divide each half into 3 equal parts. Then there are $\square$ equal parts in the whole. Each part is $\frac{1}{\square}$.

$\frac{1}{2} \div 3 = \frac{1}{\square}$.

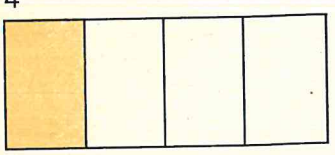
$\frac{1}{3} \div 2$



Divide each third into 2 equal parts. Then there are $\square$ equal parts in the whole. Each part is $\frac{1}{\square}$.

$\frac{1}{3} \div 2 = \frac{1}{\square}$.

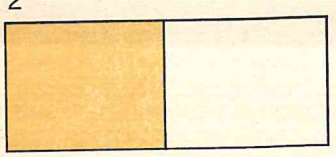
$\frac{1}{4} \div 3$



Divide each fourth into 3 equal parts. Then there are $\square$ equal parts in the whole. Each part is $\frac{1}{\square}$.

$\frac{1}{4} \div 3 = \frac{1}{\square}$.

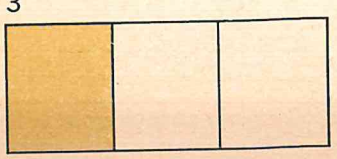
$\frac{1}{2} \div 4$



Divide each half into 4 equal parts. Then there are $\square$ equal parts in the whole. Each part is $\frac{1}{\square}$.

$\frac{1}{2} \div 4 = \frac{1}{\square}$.

$\frac{1}{3} \div 4$



Divide each third into 4 equal parts. Then there are $\square$ equal parts in the whole. Each part is $\frac{1}{\square}$.

$\frac{1}{3} \div 4 = \frac{1}{\square}$.

Compare the exercises in the first column with the exercises on page 73.

Dividing by 2 is the same as multiplying by $\frac{1}{2}$.
Dividing by 3 is the same as multiplying by $\frac{1}{3}$.
Dividing by 4 is the same as multiplying by $\frac{1}{4}$.

$2 = \frac{2}{1}$. $\frac{1}{2}$ is the reciprocal of 2.
$3 = \frac{3}{1}$. $\frac{1}{3}$ is the reciprocal of 3.
$4 = \frac{4}{1}$. $\frac{1}{4}$ is the reciprocal of 4.

To divide by a whole number, multiply by its reciprocal.

$\frac{1}{2} \div 5 = \frac{1}{5}$ of $\frac{1}{2} = \frac{1}{5} \times \frac{1}{2}$.

By the commutative law of multiplication,
$\frac{1}{5} \times \frac{1}{2} = \frac{1}{2} \times \frac{1}{5}$. So you may write:
$\frac{1}{2} \div 5 = \frac{1}{2} \times \frac{1}{5}$. Then you may use this rule:

To divide by a whole number is the same as times the reciprocal of that number.

$5 = \frac{5}{1}$. The reciprocal of 5 is $\frac{\square}{\square}$.

$6 = \frac{6}{1}$. The reciprocal of 6 is $\frac{\square}{\square}$.

$7 = \frac{7}{1}$. The reciprocal of 7 is $\frac{\square}{\square}$.

$\frac{1}{3} \div 5 = \frac{1}{3} \times \frac{1}{5} = \frac{1}{\square}$.

$\frac{1}{4} \div 5 =$

$\frac{1}{2} \div 6 = \frac{1}{2} \times \frac{1}{6} = \frac{1}{\square}$.

$\frac{1}{6} \div 5 =$

$\frac{1}{4} \div 3 = \frac{1}{4} \times \frac{1}{3} = \frac{1}{\square}$.

$\frac{1}{3} \div 4 =$

$\frac{1}{5} \div 2 = \frac{1}{5} \times \frac{\square}{\square} = \frac{\square}{\square}$.

$\frac{1}{8} \div 2 =$

$\frac{1}{4} \div 4 = \frac{1}{4} \times \frac{\square}{\square} = \frac{\square}{\square}$.

$\frac{1}{6} \div 3 =$

$\frac{1}{3} \div 3 = \frac{1}{3} \times \frac{\square}{\square} = \frac{\square}{\square}$.

$\frac{1}{4} \div 2 =$

Division by One

$\square$ $\square$ $\square$ $\square$

$1\overline{)1}$ $1\overline{)2}$ $1\overline{)3}$ $1\overline{)4}$

On page 54 you learned the law of 1:

When a number is multiplied by 1, you get the same number as product.

The exercises above suggest another law of 1:

When a number is divided by 1, you get the same number as quotient.

$7 \div 1 = 7.$ $8 \div 1 = \square.$ $9 \div 1 = \square.$

$\frac{1}{2} \div 1 = \square.$ $\frac{1}{3} \div 1 = \square.$ $\frac{1}{5} \div 1 = \square.$

$\frac{2}{3} \div 1 = \square.$ $\frac{4}{5} \div 1 = \square.$ $.3 \div 1 = \square.$

A Number Divided by Itself

$\square$ $\square$ $\square$ $\square$

$3\overline{)3}$ $4\overline{)4}$ $5\overline{)5}$ $6\overline{)6}$

When a whole number is divided by itself, the quotient is 1. $3 \div 3$ means: How many 3's are there in 3?

Answer: $\square$

$\frac{1}{2} \div \frac{1}{2}$ means: How many $\frac{1}{2}$'s are there in $\frac{1}{2}$?

Answer: $\square$

$\frac{1}{3} \div \frac{1}{3}$ means: How many $\frac{1}{3}$'s are there in $\frac{1}{3}$?

Answer: $\square$

$\frac{1}{4} \div \frac{1}{4}$ means: How many $\frac{1}{4}$'s are there in $\frac{1}{4}$?

Answer: $\square$

These are all examples of a general rule:

When a number that is not zero is divided by itself, the quotient is 1.

$\frac{2}{3} \div \frac{2}{3} = \square.$ $\frac{5}{6} \div \frac{5}{6} = \square.$ $\frac{7}{8} \div \frac{7}{8} = \square.$

$.6 \div .6 = \square.$ $.02 \div .02 = \square.$ $.43 \div .43 = \square.$

Fractions as Numerators or Denominators

$2 \div 3$ may be written as $\frac{2}{3}$. The dividend, 2, is the numerator. The divisor, 3, is the denominator.

$\frac{3}{4} \div 2$ may be written as $\frac{\frac{3}{4}}{2}$. The dividend, $\frac{3}{4}$, is the numerator. The divisor, 2, is the denominator.

$\frac{2}{5} \div \frac{1}{3}$ may be written as $\frac{\frac{2}{5}}{\frac{1}{3}}$. The dividend, $\frac{2}{5}$, is the numerator. The divisor, $\frac{1}{3}$, is the denominator.

Write in fraction form.

$\frac{2}{5} \div 6 = \frac{\frac{2}{5}}{6}.$

$\frac{4}{5} \div \frac{1}{2} =$ ——.

$\frac{2}{3} \div 4 =$ ——.

$\frac{2}{3} \div \frac{1}{3} =$ ——.

$\frac{1}{7} \div 8 =$ ——.

$\frac{3}{2} \div \frac{1}{7} =$ ——.

$\frac{4}{10} \div 2 =$ ——.

$\frac{5}{2} \div \frac{1}{5} =$ ——.

$\frac{3}{5} \div 4 =$ ——.

$\frac{1}{3} \div \frac{1}{5} =$ ——.

Find the value of each fraction below.

$\frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{2} \div \frac{1}{2} = 1.$

$\frac{\frac{1}{3}}{\frac{1}{3}} = \frac{1}{3} \div \frac{1}{3} = \square.$

$\frac{\frac{1}{4}}{\frac{1}{4}} = \frac{1}{4} \div \square = \square.$

$\frac{\frac{1}{5}}{\frac{1}{5}} = \frac{1}{5} \div \square = \square.$

$\frac{\frac{2}{3}}{\frac{2}{3}} = \frac{2}{3} \div \square = \square.$

$\frac{\frac{3}{4}}{\frac{3}{4}} = \frac{3}{4} \div \square = \square.$

Any Fraction Divided by a Whole Number

Find $\frac{3}{4} \div 2$.

Step I Write the division in fraction form.

$$\frac{3}{4} \div 2 = \frac{\frac{3}{4}}{2}.$$

Step II Find the reciprocal of the denominator. The reciprocal of 2 is $\frac{1}{2}$.

Step III Observe that $\frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{2} \div \frac{1}{2} = 1$.

Step IV Multiply $\frac{\frac{3}{4}}{2}$ by $\frac{\frac{1}{2}}{\frac{1}{2}}$.

By the law of 1, the value is not changed.

$$\frac{\frac{3}{4}}{2} \times \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{\frac{3}{4} \times \frac{1}{2}}{2 \times \frac{1}{2}} = \frac{\frac{3}{8}}{1} = \frac{3}{8} \div 1 = \frac{3}{8}.$$

Find $\frac{2}{3} \div 4$.

$\frac{2}{3} \div 4 = \frac{\frac{2}{3}}{4}$. The reciprocal of 4 is $\frac{1}{4}$.

$$\frac{\frac{2}{3}}{4} \times \frac{\frac{1}{4}}{\frac{1}{4}} = \frac{\frac{2}{3} \times \frac{1}{4}}{4 \times \frac{1}{4}} = \frac{\frac{2}{12}}{1} = \frac{2}{12} = \frac{1}{6}.$$

Find $\frac{3}{2} \div 3$.

$\frac{3}{2} \div 3 = \frac{\frac{3}{2}}{3}$. The reciprocal of 3 is $\frac{1}{3}$.

$$\frac{\frac{3}{2}}{3} \times \frac{\frac{1}{3}}{\frac{1}{3}} = \frac{\frac{3}{2} \times \frac{1}{3}}{3 \times \frac{1}{3}} = \frac{\square}{1} = \frac{\square}{\square} = \frac{\square}{\square}.$$

Find $\frac{2}{5} \div 4$.

$\frac{2}{5} \div 4 = \frac{\frac{2}{5}}{4}$. The reciprocal of 4 is $\frac{1}{4}$.

$$\frac{\frac{2}{5}}{4} \times \frac{\frac{1}{4}}{\frac{1}{4}} = \frac{\frac{2}{5} \times \frac{1}{4}}{4 \times \frac{1}{4}} = \frac{\square}{1} = \frac{\square}{\square}.$$

Find $\frac{3}{5} \div 2$.

$\frac{3}{5} \div 2 = \frac{\frac{3}{5}}{2}$. The reciprocal of 2 is $\frac{1}{2}$.

$$\frac{\frac{3}{5}}{2} \times \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{\frac{3}{5} \times \frac{1}{2}}{2 \times \frac{1}{2}} = \frac{\frac{3}{5} \times \frac{1}{2}}{1} = \frac{3}{5} \times \frac{1}{2} = \frac{\square}{\square}.$$

Find $\frac{2}{7} \div 3$.

$\frac{2}{7} \div 3 = \frac{\frac{2}{7}}{3}$. The reciprocal of 3 is $\frac{1}{3}$.

$$\frac{\frac{2}{7}}{3} \times \frac{\frac{1}{3}}{\frac{1}{3}} = \frac{\frac{2}{7} \times \frac{1}{3}}{3 \times \frac{1}{3}} = \frac{\frac{2}{7} \times \frac{1}{3}}{1} = \frac{2}{7} \times \frac{1}{3} = \frac{\square}{\square}.$$

The steps marked in color show you may follow the rule stated in the second column of page 80: To divide by a whole number is the same as times the reciprocal of that number.

Use the rule in these exercises:

$\frac{3}{8} \div 2 = \frac{3}{8} \times \frac{1}{2} = \frac{\square}{\square}$. $\frac{5}{3} \div 2 =$

$\frac{2}{5} \div 3 = \frac{2}{5} \times \frac{\square}{\square} = \frac{\square}{\square}$. $\frac{8}{5} \div 2 =$

$\frac{2}{3} \div 2 = \frac{2}{3} \times \frac{\square}{\square} = \frac{\square}{\square}$. $\frac{4}{3} \div 3 =$

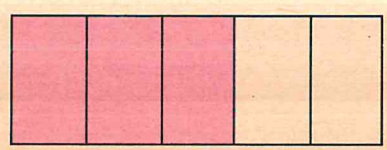
$\frac{3}{5} \div 4 = \frac{3}{5} \times \frac{\square}{\square} = \frac{\square}{\square}$. $\frac{5}{4} \div 4 =$

$\frac{2}{9} \div 4 = \frac{2}{9} \times \frac{\square}{\square} = \frac{\square}{\square}$. $\frac{3}{10} \div 2 =$

$\frac{1}{2} \div 3 = \frac{1}{2} \times \frac{\square}{\square} = \frac{\square}{\square}$. $\frac{1}{4} \div 2 =$

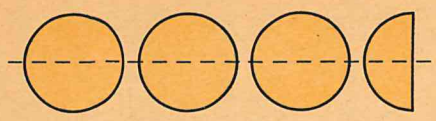
$\frac{1}{6} \div 2 = \frac{1}{6} \times \frac{\square}{\square} = \frac{\square}{\square}$. $\frac{1}{5} \div 2 =$

$\frac{3}{5} \div 2 = \frac{3}{5} \times \frac{\square}{\square} = \frac{\square}{\square}$.



Check your answer by dividing each fifth into 2 equal parts.

A Mixed Number Divided by a Whole Number



$3\frac{1}{2} \div 2$

Tom and Bill have $3\frac{1}{2}$ cookies that they will share equally. How much does each one get? The diagram shows that the answer is:

$(3 \div 2) + \left(\frac{1}{2} \div 2\right)$. $\quad 3 \div 2 = \frac{3}{2}$; $\quad \frac{1}{2} \div 2 = \frac{1}{4}$.

$\frac{3}{2} + \frac{1}{4} = \frac{\square}{4} + \frac{1}{4} = \frac{\square}{4} = \square\frac{\square}{4}$.

Answer: Each one gets $\square\frac{\square}{4}$ cookies.

Find $4\frac{1}{3} \div 2$.

$4 \div 2 = 2 \longrightarrow 2$

$\frac{1}{3} \div 2 = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \longrightarrow + \frac{1}{6}$

Answer: $2\frac{1}{6}$

Find $5\frac{1}{2} \div 4$.

$5 \div 4 = 1\frac{1}{4} \longrightarrow \square\frac{\square}{\square}$

$\frac{1}{2} \div 4 = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} \longrightarrow + \frac{\square}{\square}$

Find $6\frac{2}{3} \div 3$.

$6 \div 3 = \longrightarrow \square$

$\frac{2}{3} \div 3 = \frac{2}{3} \times \frac{1}{\square} = \frac{\square}{\square} \longrightarrow + \frac{\square}{\square}$

Find $2\frac{1}{2} \div 3$.

$2 \div 3 =$

$\frac{1}{2} \div 3 =$

Find $3\frac{1}{4} \div 4$.

Find $4\frac{1}{2} \div 3$. If you change the mixed number to an improper fraction, you can then divide by the method of page 82.

$4\frac{1}{2} \div 3 = \frac{9}{2} \div 3 = \frac{9}{2} \times \frac{1}{3} = \frac{9}{6} = 1\frac{3}{6} = 1\frac{1}{2}$.

Change to halves.

$3 = \frac{\square}{2}$. $\quad 5 = \frac{\square}{2}$.

$2 = \frac{\square}{2}$. $\quad 6 = \frac{\square}{2}$.

Change to thirds.

$3 = \frac{\square}{3}$. $\quad 5 = \frac{\square}{3}$.

$2 = \frac{\square}{3}$. $\quad 6 = \frac{\square}{3}$.

Change to fourths.

$3 = \frac{\square}{4}$. $\quad 5 = \frac{\square}{4}$.

$2 = \frac{\square}{4}$. $\quad 6 = \frac{\square}{4}$.

Change to fifths.

$3 = \frac{\square}{5}$. $\quad 5 = \frac{\square}{5}$.

$2 = \frac{\square}{5}$. $\quad 6 = \frac{\square}{5}$.

Change to improper fractions.

$3\frac{1}{2} = \frac{\square}{2} + \frac{1}{2} = \frac{\square}{2}$. $\quad 5\frac{2}{3} = \frac{\square}{3} + \frac{2}{3} = \frac{\square}{3}$.

Short Method Multiply the whole number by the denominator. Then add the numerator. The result is the numerator of the improper fraction.

$5\frac{1}{2} = \frac{\square}{2}$. $\quad 6\frac{1}{3} = \frac{\square}{3}$. $\quad 7\frac{3}{4} = \frac{\square}{4}$.

Change the mixed numbers to improper fractions. Then divide.

$2\frac{1}{2} \div 3 = \frac{\square}{2} \div 3 = \frac{\square}{2} \times \frac{\square}{\square} =$

$4\frac{1}{4} \div 5 = \frac{\square}{4} \div 5 = \frac{\square}{4} \times \frac{\square}{\square} =$

$3\frac{1}{3} \div 2 = \frac{\square}{\square} \div 2 = \frac{\square}{\square} \times \frac{\square}{\square} =$

$5\frac{1}{3} \div 4 = \frac{\square}{\square} \div 4 = \frac{\square}{\square} \times \frac{\square}{\square} =$

$6\frac{1}{2} \div 2 =$

$3\frac{1}{4} \div 3 =$

$3\frac{1}{5} \div 4 =$

$2\frac{1}{2} \div 5 =$

A Decimal Times a Decimal

Multiply .3 × .7.

$.3 \times .7 = \frac{3}{10} \times \frac{7}{10} = \frac{21}{100} = .21.$

.3 — one decimal place; .7 — one decimal place; .21 — two decimal places

Multiply .2 × .04.

$.2 \times .04 = \frac{2}{10} \times \frac{4}{100} = \frac{8}{1000} = .\square\square\square.$

.2 — one decimal place; .04 — two decimal places; three decimal places

Multiply 2.1 × .5.

$2.1 \times .5 = 2\frac{1}{10} \times \frac{5}{10} = \frac{21}{10} \times \frac{5}{10} = \frac{\square\square\square}{100} = \square.\square\square.$

2.1 — one decimal place; .5 — one decimal place; two decimal places

When two decimals are multiplied, the number of decimal places in the product is the sum of the number of decimal places in the multiplier and the number of decimal places in the multiplicand.

To multiply .6 × 3.2, first multiply 32 by 6.

Then put the decimal point 2 places from the right.

1 decimal place → 3 . 2
1 decimal place → × . 6
2 decimal places → □ . □ □

Multiply.

4.2 ← 1 decimal place
×.8 ← 1 decimal place
← 2 decimal places

2.15 ← 2 decimal places
×.4 ← 1 decimal place
← 3 decimal places

3.6 ← 1 decimal place
×.02 ← 2 decimal places
← □ decimal places

1.02 ← 2 decimal places
×.3 ← 1 decimal place
← □ decimal places

Multiply in decimal form. Then check by multiplying in fraction form.

.8
×.3
.24

Check. $.8 = \frac{8}{10}$. $.3 = \frac{3}{10}$.

$\frac{3}{10} \times \frac{8}{10} = \frac{24}{100} = .24.$

.5
×.6
□

Check. $.5 = \frac{\square}{\square}$. $.6 = \frac{\square}{\square}$.

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} =$

1.2
×.04
□

Check. $1.2 = \frac{\square}{\square}$. $.04 = \frac{\square}{\square}$.

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} =$

2.3
×.07
□

Check. $2.3 = \frac{\square}{\square}$. $.07 = \frac{\square}{\square}$.

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} =$

1.2
×2.4
□ □
□ □ □
□ □

Check. $1.2 = \frac{\square}{\square}$. $2.4 = \frac{\square}{\square}$.

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} =$

2.5
×3.1

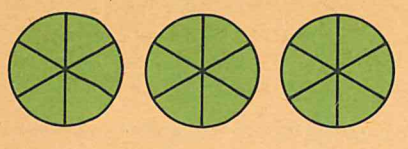
Check.

$2.50
×.6
□

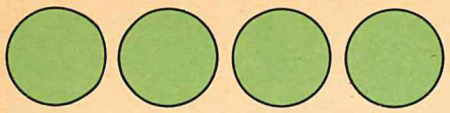
The retail price of a book is $2.50. A book-dealer pays .6 of the retail price of the book. How much does he pay? (Hint: $.500 = \frac{500}{1000} = \frac{50}{100} = .50.$)

Answer: □

A Whole Number Divided by a Unit Fraction



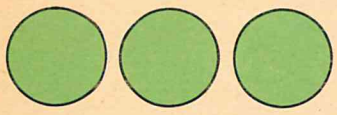
3 pies are cut into slices. Each slice is $\frac{1}{6}$ of a pie. How many slices are there? The question is like asking: How many sixths are there in 3? This is like asking for $3 \div \frac{1}{6}$. The diagram shows that the answer is $3 \times 6 = \square$.



Find $4 \div \frac{1}{3}$.

Cut each whole into thirds, and count the thirds.

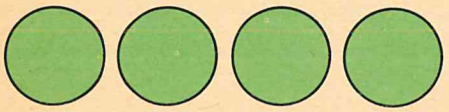
$4 \div \frac{1}{3} = 4 \times \square = \square$.



Find $3 \div \frac{1}{2}$.

Cut each whole into halves, and count the halves.

$3 \div \frac{1}{2} = 3 \times \square = \square$.



Find $4 \div \frac{1}{4}$.

Cut each whole into fourths, and count the fourths.

$4 \div \frac{1}{4} = 4 \times \square = \square$.

The reciprocal of $\frac{1}{4}$ is $\frac{4}{1} = 4$.

The reciprocal of $\frac{1}{2}$ is $\frac{2}{1} = \square$.

The reciprocal of $\frac{1}{3}$ is $\frac{\square}{\square} = \square$.

The reciprocal of $\frac{1}{5}$ is $\frac{\square}{\square} = \square$.

The reciprocal of $\frac{1}{6}$ is $\frac{\square}{\square} = \square$.

Find $5 \div \frac{1}{2}$, by first writing it in fraction form.

$5 \div \frac{1}{2} = \frac{5}{\frac{1}{2}}$.

The reciprocal of $\frac{1}{2}$ is 2.
$\frac{2}{2} = 1$.

If you multiply by $\frac{2}{2}$, the value of the fraction is unchanged (law of 1).

$\frac{5}{\frac{1}{2}} \times \frac{2}{2} = \frac{5 \times 2}{\frac{1}{2} \times 2} = \frac{10}{1} = 10$.

Divide, using the fraction form and the law of 1.

$6 \div \frac{1}{3} = \frac{6}{\frac{1}{3}} = \frac{6}{\frac{1}{3}} \times \frac{3}{3} = \frac{\square}{1} = \square$.

$3 \div \frac{1}{4} = \frac{3}{\frac{1}{4}} = \frac{3}{\frac{1}{4}} \times \frac{4}{4} = \frac{\square}{1} = \square$.

$5 \div \frac{1}{2} = \frac{5}{\frac{1}{2}} = \frac{5}{\frac{1}{2}} \times \frac{2}{2} = \frac{\square}{1} = \square$.

$2 \div \frac{1}{3} =$

$3 \div \frac{1}{3} =$

The exercises done on this page show that

$3 \div \frac{1}{6} = 3 \times 6$; $4 \div \frac{1}{3} = 4 \times 3$; $3 \div \frac{1}{2} = 3 \times 2$.

They suggest the rule:

To divide by a unit fraction is the same as times the reciprocal of that fraction.

Use the rule to do these exercises.

$8 \div \frac{1}{2} = 8 \times 2 = \square$.

$7 \div \frac{1}{4} = 7 \times \square = \square$.

$4 \div \frac{1}{5} = 4 \times \square = \square$.

$6 \div \frac{1}{2} = 6 \times \square = \square$.

$9 \div \frac{1}{3} = 9 \times \square = \square$.

$10 \div \frac{1}{2} = 10 \times \square = \square$.

$5 \div \frac{1}{8} = 5 \times \square = \square$.

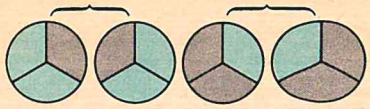
$8 \div \frac{1}{4} =$

$9 \div \frac{1}{6} =$

$7 \div \frac{1}{7} =$

A Whole Number Divided by a Fraction

Find $4 \div \frac{2}{3}$. First Way



Divide each of 4 wholes into thirds. Group the thirds in sets of 2. There are 6 sets, each containing 2 thirds, in 4.

So $4 \div \frac{2}{3} = 6$.

Second Way The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$4 \div \frac{2}{3} = \frac{4}{\frac{2}{3}}$. Multiply by $\frac{\frac{3}{2}}{\frac{3}{2}}$.

$$\frac{4}{\frac{2}{3}} \times \frac{\frac{3}{2}}{\frac{3}{2}} = \frac{4 \times \frac{3}{2}}{\frac{2}{3} \times \frac{3}{2}} = \frac{4 \times \frac{3}{2}}{1} = 4 \times \frac{3}{2} = 6.$$

The steps that are in color show that you can extend the rule of page 85 to division by any fraction:

To divide by a fraction is the same as times the reciprocal of that fraction.

Use the rule to do these exercises.

$4 \div \frac{3}{2} = 4 \times \frac{2}{3} = \frac{\square}{3}$.	$14 \div \frac{1}{2} =$
$5 \div \frac{2}{5} = 5 \times \frac{\square}{\square} = \frac{\square}{\square}$.	$6 \div \frac{3}{5} =$
$6 \div \frac{3}{2} = 6 \times \frac{\square}{\square} = \frac{\square}{\square} = \square$.	$8 \div \frac{2}{3} =$
$8 \div \frac{4}{5} = 8 \times \frac{\square}{\square} = \frac{\square}{\square} = \square$.	$9 \div \frac{6}{10} =$
$9 \div \frac{3}{4} = 9 \times \frac{\square}{\square} = \frac{\square}{\square} = \square$.	$10 \div \frac{1}{10} =$
$2 \div \frac{2}{5} = 2 \times \frac{\square}{\square} = \frac{\square}{\square} = \square$.	$4 \div \frac{4}{5} =$
$\frac{3}{2} \div 2 = \frac{3}{2} \times \frac{\square}{\square} = \frac{\square}{\square}$.	$\frac{2}{5} \div 4 =$
$\frac{4}{5} \div 3 = \frac{4}{5} \times \frac{\square}{\square} = \frac{\square}{\square}$.	$\frac{2}{3} \div 3 =$
$6\frac{1}{2} \div 3 = \frac{\square}{\square} \div 3 =$	
$2\frac{1}{3} \div 7 = \frac{\square}{\square} \div 7 =$	

Review

$\frac{2}{3} \times \frac{3}{2} = \square$.	$\frac{5}{6} \times \frac{6}{5} = \square$.
$\frac{3}{4} \times \frac{\square}{\square} = 1$.	$\frac{2}{9} \times \frac{\square}{\square} = 1$.
$\frac{2}{5} \times \frac{\square}{\square} = 1$.	$\frac{3}{8} \times \frac{\square}{\square} = 1$.
$2 \times \frac{\square}{\square} = 1$.	$4 \times \frac{\square}{\square} = 1$.

The reciprocal of $\frac{1}{3}$ is $\square$.

The reciprocal of $\frac{1}{5}$ is $\square$.

The reciprocal of $\frac{1}{10}$ is $\square$.

The reciprocal of 3 is $\frac{\square}{\square}$.

The reciprocal of 2 is $\frac{\square}{\square}$.

The reciprocal of $\frac{4}{5}$ is $\frac{\square}{\square}$.

The reciprocal of $\frac{3}{7}$ is $\frac{\square}{\square}$.

Write in the box next to each statement below the abbreviation that shows which law it is an example of. Use these abbreviations:

CM: Commutative Law of Multiplication
AM: Associative Law of Multiplication
1: The Law of One
D: The Distributive Law

$\left(3 \times \frac{1}{2}\right) \times 6 = 3 \times \left(\frac{1}{2} \times 6\right)$. $\square$

$\frac{3}{\frac{1}{2}} = \frac{3}{\frac{1}{2}} \times 2$. $\square$

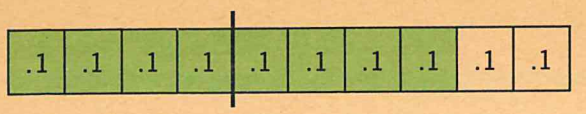
$4 \times (3\frac{1}{2}) = (4 \times 3) + (4 \times \frac{1}{2})$. $\square$

$5.64 \times 8 = 8 \times 5.64$. $\square$

A Decimal Fraction Divided by a Whole Number

Find .8 ÷ 2.

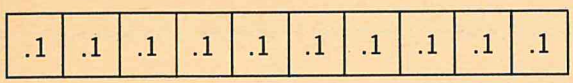
Remember that .8 = 8 tenths = $\frac{8}{10}$.



Think: If 8 tenths are divided into 2 equal groups, how many tenths are there in each group?

$\begin{array}{r} 4 \text{ tenths} \\ 2\overline{)8 \text{ tenths}} \end{array}$ $\frac{8}{10} \div 2 = \frac{4}{10}$. $\begin{array}{r} .4 \\ 2\overline{).8} \end{array}$

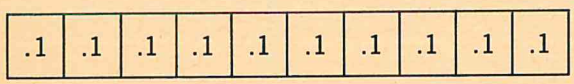
Find .6 ÷ 2.



Color 6 tenths. Then draw a line that divides them into 2 equal groups.

$\begin{array}{r} \square \text{ tenths} \\ 2\overline{)6 \text{ tenths}} \end{array}$ $\frac{6}{10} \div 2 = \frac{\square}{10}$. $\begin{array}{r} .\square \\ 2\overline{).6} \end{array}$

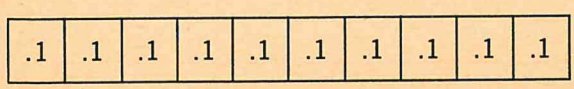
Find .9 ÷ 3.



Color 9 tenths. Then draw lines that divide them into 3 equal groups.

$\begin{array}{r} \square \text{ tenths} \\ 3\overline{)9 \text{ tenths}} \end{array}$ $\frac{9}{10} \div 3 = \frac{\square}{10}$. $\begin{array}{r} .\square \\ 3\overline{).9} \end{array}$

Find .8 ÷ 4.



Color 8 tenths. Then draw lines that divide them into 4 equal groups.

$\begin{array}{r} \square \text{ tenths} \\ 4\overline{)8 \text{ tenths}} \end{array}$ $\frac{8}{10} \div 4 = \frac{\square}{10}$. $\begin{array}{r} .\square \\ 4\overline{).8} \end{array}$

$2\overline{).4}$	$3\overline{).3}$	$1\overline{).5}$
$2\overline{).2}$	$4\overline{).4}$	$3\overline{).6}$

Find .12 ÷ 3.

Think: If 12 hundredths are divided into 3 equal groups, there are 4 hundredths in each group.

$\begin{array}{r} 4 \text{ hundredths} \\ 3\overline{)12 \text{ hundredths}} \end{array}$ $\frac{12}{100} \div 3 = \frac{4}{100}$. $\begin{array}{r} .04 \\ 3\overline{).12} \end{array}$

Find .25 ÷ 5.

Think: If 25 hundredths are divided into 5 equal groups, there are $\square$ hundredths in each group.

$\begin{array}{r} \square \text{ hundredths} \\ 5\overline{)25 \text{ hundredths}} \end{array}$ $\frac{25}{100} \div 5 = \frac{\square}{100}$. $\begin{array}{r} .\square\square \\ 5\overline{).25} \end{array}$

Find .16 ÷ 4.

$\begin{array}{r} \square \text{ hundredths} \\ 4\overline{)16 \text{ hundredths}} \end{array}$ $4\overline{).16}$

Find .18 ÷ 9.

$\begin{array}{r} \square \text{ hundredths} \\ 9\overline{)18 \text{ hundredths}} \end{array}$ $\frac{18}{100} \div 9 = \frac{\square}{100}$. $9\overline{).18}$

Find .36 ÷ 3.

Think: 36 hundredths divided into 3 equal parts give $\square$ hundredths in each part.

$\begin{array}{r} \square \text{ hundredths} \\ 3\overline{)36 \text{ hundredths}} \end{array}$ $\frac{36}{100} \div 3 = \frac{\square}{100}$. $3\overline{).36}$

Or think: 3 tenths divided into 3 equal parts gives $\square$ tenths in each part. 6 hundredths divided into 3 equal parts gives $\square$ hundredths in each part.

Find .96 ÷ 4.

9 tenths ÷ 4 = 2 tenths and 1 tenth left. Exchange 1 tenth for 10 hundredths. 16 hundredths ÷ 4 = 4 hundredths.

$\begin{array}{r} .24 \\ 4\overline{).96} \end{array}$

$5\overline{).15}$	$7\overline{).21}$	$6\overline{).18}$
$2\overline{).48}$	$3\overline{).69}$	$4\overline{).84}$
$4\overline{).56}$	$5\overline{).75}$	$6\overline{).72}$
$2\overline{).34}$	$3\overline{).48}$	$4\overline{).64}$

A Mixed Decimal Divided by a Whole Number

Find 6.9 ÷ 3.

Think: 6 wholes divided into 3 equal parts gives ☐ wholes in each part. 9 tenths divided into 3 equal parts gives ☐ tenths in each part.

$$3\overline{)6.9}$$

Find 4.68 ÷ 2.

4 wholes ÷ 2 = ☐ wholes.

6 tenths ÷ 2 = ☐ tenths.

8 hundredths ÷ 2 = ☐ hundredths.

$$2\overline{)4.68}$$

Find 8.4 ÷ 4.

8 wholes ÷ 4 = ☐ wholes.

4 tenths ÷ 4 = ☐ tenths.

$$4\overline{)8.4}$$

Find 9.36 ÷ 3.

9 wholes ÷ 3 = ☐ wholes.

3 tenths ÷ 3 = ☐ tenths.

6 hundredths ÷ 3 = ☐ hundredths.

$$3\overline{)9.36}$$

Find 8.46 ÷ 2.

8 wholes ÷ 2 = ☐ wholes.

4 tenths ÷ 2 = ☐ tenths.

6 hundredths ÷ 2 = ☐ hundredths.

$$2\overline{)8.46}$$

Find 3.09 ÷ 3.

3 wholes ÷ 3 = ☐ whole.

0 tenths ÷ 3 = ☐ tenths.

9 hundredths ÷ 3 = ☐ hundredths.

$$3\overline{)3.09}$$

Find 6.2 ÷ 2.

6 wholes ÷ 2 = ☐ wholes.

2 tenths ÷ 2 = ☐ tenths.

$$2\overline{)6.2}$$

$3\overline{)9.6}$	$2\overline{)4.4}$	$4\overline{)4.8}$
$2\overline{)8.24}$	$4\overline{)8.08}$	$3\overline{)6.03}$
$3\overline{)6.69}$	$2\overline{)2.48}$	$4\overline{)4.44}$
$4\overline{)8.04}$	$3\overline{)9.63}$	$2\overline{)4.82}$

Find 2.4 ÷ 3.

Estimate: 2.4 is less than 3. Then the quotient is less than 1.

Think: In dividing 2.4 into 3 equal parts, there are not enough wholes to give 1 whole to each part. Exchange 2 wholes for 20 tenths.

24 tenths ÷ 3 = ☐ tenths.

$$3\overline{)2.4}$$

Find 9.28 ÷ 4.

Estimate: 9.28 is more than 8. Then the quotient is more than 2.

Think: 9 ÷ 4 = 2 and 1 left. Exchange 1 whole for 10 tenths.

12 tenths ÷ 4 = ☐ tenths.

8 hundredths ÷ 4 = ☐ hundredths.

$$4\overline{)9.28}$$

Find 7.6 ÷ 2.

Estimate: more than ☐

$$2\overline{)7.6}$$

Find 5.6 ÷ 4.

Estimate: more than ☐

$$4\overline{)5.6}$$

Find 8.4 ÷ 3.

Estimate: more than ☐

$$3\overline{)8.4}$$

Find 9.6 ÷ 2.

Estimate: more than ☐

$$2\overline{)9.6}$$

Find 1.84 ÷ 2.

Estimate: more than ☐

$$2\overline{)1.84}$$

Find 5.46 ÷ 2.

Estimate: more than ☐

$$2\overline{)5.46}$$

Find 7.53 ÷ 3.

Estimate: more than ☐

$$3\overline{)7.53}$$

Find 3.68 ÷ 4.

Estimate: more than ☐

$$4\overline{)3.68}$$

Find 4.38 ÷ 3.

$$3\overline{)4.38}$$

Find 7.76 ÷ 4.

$$4\overline{)7.76}$$

A Fraction Divided by a Unit Fraction

Find $\frac{3}{4} \div \frac{1}{4}$.

First Way Ask: How many fourths are there in 3 fourths?

Answer: $\square$

Second Way First write $\frac{3}{4} \div \frac{1}{4} = \frac{\frac{3}{4}}{\frac{1}{4}}$.

The reciprocal of $\frac{1}{4}$ is 4. Multiply by $\frac{4}{4}$.

$$\frac{\frac{3}{4}}{\frac{1}{4}} \times \frac{4}{4} = \frac{\frac{3}{4} \times 4}{\frac{1}{4} \times 4} = \frac{\frac{3}{4} \times 4}{1} = \frac{3}{4} \times 4 = \frac{12}{4} = \square.$$

The steps that are in color show that you can use the rule of page 85:

To divide by a unit fraction is the same as times the reciprocal of that fraction.

Use the rule to do the following exercises.

$\frac{6}{9} \div \frac{1}{3} = \frac{6}{9} \times 3 = \frac{18}{9} = \square$.

$\frac{3}{5} \div \frac{1}{2} = \frac{3}{5} \times \square = \frac{\square}{5} = \square\frac{\square}{5}$.

$\frac{4}{10} \div \frac{1}{5} = \frac{4}{10} \times \square = \frac{\square}{10} = \square$.

$\frac{5}{8} \div \frac{1}{4} = \frac{5}{8} \times \square = \frac{\square}{8} = \square\frac{\square}{8} = \square\frac{\square}{\square}$.

$\frac{3}{4} \div \frac{1}{2} =$

$\frac{5}{2} \div \frac{1}{4} =$

$\frac{7}{10} \div \frac{1}{5} =$

$\frac{9}{4} \div \frac{1}{4} =$

$\frac{2}{3} \div \frac{1}{2} =$

$\frac{2}{3} \div \frac{1}{3} =$

$\frac{3}{4} \div \frac{1}{8} =$

A Fraction Divided by a Fraction

Find $\frac{8}{3} \div \frac{2}{3}$.

First Way

Group 8 thirds in sets of 2 thirds each. There are $\square$ sets.

$\frac{8}{3} \div \frac{2}{3} = \square$

Second Way First write $\frac{8}{3} \div \frac{2}{3} = \frac{\frac{8}{3}}{\frac{2}{3}}$.

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. Multiply by $\frac{\frac{3}{2}}{\frac{3}{2}}$.

$$\frac{\frac{8}{3}}{\frac{2}{3}} \times \frac{\frac{3}{2}}{\frac{3}{2}} = \frac{\frac{8}{3} \times \frac{3}{2}}{\frac{2}{3} \times \frac{3}{2}} = \frac{\frac{8}{3} \times \frac{3}{2}}{1} = \frac{8}{3} \times \frac{3}{2} = \frac{24}{6} = \square.$$

The steps that are in color show that you may use the rule of page 86: To divide by a fraction is the same as times the reciprocal of that fraction.

Use the rule to do the following exercises.

$\frac{4}{5} \div \frac{2}{5} = \frac{4}{5} \times \frac{5}{2} = \frac{20}{10} = \square$.

$\frac{7}{10} \div \frac{3}{2} = \frac{7}{10} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$.

$\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square} = \square\frac{\square}{\square}$.

$\frac{2}{3} \div \frac{5}{6} =$

$\frac{1}{8} \div \frac{3}{4} =$

$\frac{1}{2} \div \frac{5}{2} =$

$\frac{1}{4} \div \frac{2}{3} =$

$\frac{1}{6} \div \frac{3}{2} =$

$\frac{5}{3} \div \frac{5}{2} =$

Multiplying a Decimal by Ten

$10 \times .2 = 10 \times \frac{2}{10} = \frac{20}{10} = 2.$

$10 \times .3 = 10 \times \frac{3}{10} = \frac{\square}{10} = \square.$

$10 \times .4 = 10 \times \frac{4}{10} = \frac{\square}{10} = \square.$

$10 \times .5 = 10 \times \frac{5}{10} = \frac{\square}{10} = \square.$

	Units		Tenths
10 ×		.	2
=	2	.	
10 ×		.	3
=	3	.	
10 ×		.	4
=	4	.	
10 ×		.	5
=	5	.	

The exercises done above are shown again in the chart at the left. As you found on page 6, multiplying a number by 10 moves each digit one place to the left.

In these cases, a digit in the tenths place is moved to the units place. The number of tenths is changed to a number of units.

$10 \times .6 = 6.$

$10 \times .8 = \square.$

$10 \times .7 = \square.$

$10 \times .9 = \square.$

$.1 \times 10 = 10 \times .1 = \square.$

$.3 \times 10 = 10 \times .3 = \square.$

$.5 \times 10 = 10 \times .5 = \square.$

$.7 \times 10 = 10 \times .7 = \square.$

$.2 \times 10 = 10 \times .2 = \square.$

$.4 \times 10 = 10 \times .4 = \square.$

$.6 \times 10 = 10 \times .6 = \square.$

$.8 \times 10 = 10 \times .8 = \square.$

	Tens		Units		Tenths
10 ×			2	.	3
=	2		3	.	
10 ×			4	.	6
=	4		6	.	

In the chart at the left, you use the same rule to multiply a mixed decimal by 10.

$10 \times 2.3 = 23.$

$10 \times 4.6 = 46.$

$10 \times 3.5 = \square.$

$10 \times 2.9 = \square.$

$8.4 \times 10 = \square.$

$7.2 \times 10 = \square.$

	Units		Tenths		Hundredth
10 ×		.	6		2
=	6	.	2		

To multiply a number of hundredths by 10, move each digit one place to the left.

$10 \times .62 = 6.2.$

$10 \times .45 = \square\square.$

$10 \times .37 = \square.$

$10 \times .24 = \square.$

$10 \times .29 = \square\square.$

$10 \times .86 = \square.$

$10 \times .71 = \square.$

Another Quick Way to Multiply by Ten

To multiply a number by 10, instead of moving each digit one place to the left, **move the decimal point one place to the right.**

Example $10 \times .6 = 6.$

Old position of decimal point — New position

Example $10 \times .35 = 3.5$

Multiply by 10 by moving the decimal point one place to the right. Be sure to show the decimal point in the product.

$10 \times .2 = \square.$

$10 \times .36 = \square.$

$10 \times .235 = \square.$

$10 \times 2.4 = \square.$

$10 \times .9 = \square.$

$10 \times .72 = \square.$

$10 \times .471 = \square.$

$10 \times 5.6 = \square.$

$.7 \times 10 = 10 \times .7 = \square.$

$2.3 \times 10 = 10 \times 2.3 = \square.$

$3.85 \times 10 = 10 \times 3.85 = \square.$

$4.8 \times 10 = 10 \times 4.8 = \square.$

A Decimal Fraction Divided by Tenths

Find .6 ÷ .2.

First write the division in fraction form: $.6 \div .2 = \frac{.6}{.2}$.

You can make the denominator a whole number by multiplying it by 10.

To leave the value of the fraction unchanged, multiply it by $\frac{10}{10}$.

$$\frac{.6}{.2} \times \frac{10}{10} = \frac{.6 \times 10}{.2 \times 10} = \frac{6}{2} = 3.$$

Find .8 ÷ .4.

$$\frac{.8}{.4} = \frac{.8}{.4} \times \frac{10}{10} = \frac{.8 \times 10}{.4 \times 10} = \frac{8}{4} = \square.$$

Find .9 ÷ .3.

$$\frac{.9}{.3} = \frac{.9}{.3} \times \frac{10}{10} = \frac{.9 \times 10}{.3 \times 10} = \frac{9}{3} = \square.$$

The last exercise above can be written as shown here.

$$.3\overline{).9}$$

$$3.\ \square\overline{)9.}$$

To multiply the divisor and the dividend by 10, move the decimal points one place to the right. The new divisor is a whole number. Then divide.

Multiply the divisor and dividend by 10 by moving the decimal points one place to the right. Rewrite the new divisor and dividend. Then divide.

$$.4\overline{).8}$$

$$4.\ \square\overline{)8.}$$

$$.2\overline{).8}$$

$$.3\overline{).6}$$

$$.2\overline{).4\,6}$$

$$2.\ \overline{)4.6}$$

$$.3\overline{).15}$$

$$.4\overline{).28}$$

A Mixed Decimal Divided by Tenths

Find 2.4 ÷ .3.

$$\frac{2.4}{.3} = \frac{2.4}{.3} \times \frac{10}{10} = \frac{2.4 \times 10}{.3 \times 10} = \frac{24}{3} = \square.$$

Find 5.6 ÷ .4.

$$\frac{5.6}{.4} = \frac{5.6}{.4} \times \frac{10}{10} = \frac{5.6 \times 10}{.4 \times 10} = \frac{\square}{\square} = \square.$$

Find 3.2 ÷ .8.

$$\frac{3.2}{.8} = \frac{3.2}{.8} \times \frac{\square}{\square} = \frac{3.2 \times \square}{.8 \times \square} = \frac{\square}{\square} = \square.$$

Short Way to Divide by Tenths: If the divisor has one decimal place, move the decimal point in the divisor and in the dividend one place to the right. This is like multiplying by $\frac{10}{10}$ or 1. Rewrite the exercise with the new divisor and dividend. Then divide.

$$.3\overline{).4.8}$$

$$3.\ \square\overline{)4\ 8.}$$

$$.5\overline{)8.5}$$

$$.8\overline{)6.4}$$

$$.2\overline{)2.4\,6}$$

$$2)\overline{2\ 4.6}$$

$$.4\overline{)4.8\ 4}$$

$$.7\overline{)1.4\ 7}$$

$$.9\overline{)8.1}$$

$$.7\overline{)5.6}$$

$$.2\overline{)7.8}$$

$$.6\overline{)3.6}$$

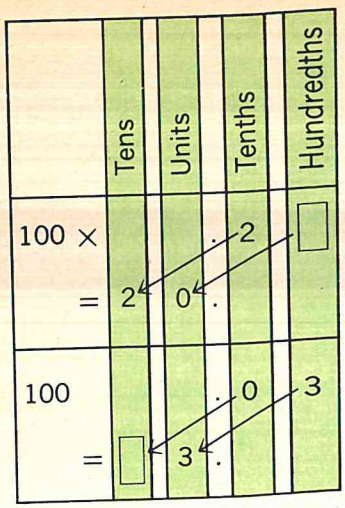
$$.5\overline{)6.5}$$

$$.4\overline{)9.2}$$

Multiplying a Decimal by One Hundred

$100 \times .2 = (10 \times 10) \times .2$
$= 10 \times (10 \times .2)$
$= 10 \times 2$
$= 20.$

$100 \times .03 = (10 \times 10) \times .03$
$= 10 \times (10 \times .03)$
$= 10 \times .3$
$= 3.$



Multiplying by 100 is like multiplying by 10 twice. Multiplying by 10 moves each digit one place to the left. Multiplying by 100 moves each digit two places to the left. This has the same effect as moving the decimal point two places to the right, and filling in an empty place with 0 as a place holder.

Example 1 0 0 × .2 = 2 0.

Old position of decimal point — New position

Example 1 0 0 × .0 3 = 3.

Multiply by 100 by moving the decimal point two places to the right. If necessary, fill in empty places with 0 as a place holder.

$100 \times .4 = 40.$

$100 \times .6 = \square.$

$100 \times .9 = \square.$

$100 \times 3.2 = 320.$

$100 \times 4.7 = \square.$

$100 \times 6.5 = \square.$

$100 \times .07 = 7.$

$100 \times .08 = \square.$

$100 \times .01 = \square.$

$100 \times 2.04 = 204.$

$100 \times 3.75 = \square.$

$100 \times 5.99 = \square.$

$.06 \times 100 = 100 \times .06 = \square.$

$2.65 \times 100 = 100 \times 2.65 = \square.$

$1.8 \times 100 = 100 \times 1.8 = \square.$

$.73 \times 100 = 100 \times .73 = \square.$

A Decimal Divided by Hundredths

Find $.8 \div .02$.

First write the division in fraction form.

$.8 \div .02 = \frac{.8}{.02}$. You can make the denominator a whole number by multiplying it by 100. **To leave the value of the fraction unchanged, multiply by $\frac{100}{100}$.**

$$\frac{.8}{.02} \times \frac{100}{100} = \frac{.8 \times 100}{.02 \times 100} = \frac{80}{2} = 40.$$

Find $.25 \div .05$.

$$\frac{.25}{.05} = \frac{.25}{.05} \times \frac{100}{100} = \frac{.25 \times 100}{.05 \times 100} = \frac{\square}{\square} = \square.$$

Find $8.32 \div .04$.

$$\frac{8.32}{.04} = \frac{8.32}{.04} \times \frac{100}{100} = \frac{8.32 \times 100}{.04 \times 100} = \frac{832}{4} = \square.$$

The last example above can be written as shown here.

$.04\overline{)8.32}$

$\square\square\square.$

$4\overline{)8\ 3\ 2.}$

To multiply the divisor and the dividend by 100, move the decimal points two places to the right. The new divisor is a whole number. Then divide.

Multiply the divisor and dividend by 100 by moving the decimal point two places to the right. Rewrite the new divisor and dividend. Then divide.

$.04\overline{).12}$

$\square$

$4\overline{)12}$

$.06\overline{).24}$

$.07\overline{).49}$

$.02\overline{).8}$

$\square$

$2\overline{)80}$

$.03\overline{).9}$

$.04\overline{).8}$

$.06\overline{)2.46}$

$6\overline{)246}$

$.05\overline{)3.75}$

$.04\overline{)4.12}$

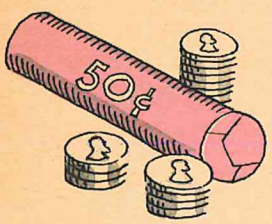
Dividing a Sum of Money by a Number of Cents

If a candy bar costs $.05, how many bars can be bought for $.75? To find how many .05's there are in .75, you divide.

$.05\overline{).75}$ $\qquad$ $5\overline{)7\ 5}$

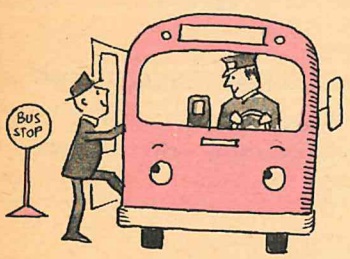
If you save $.06 a week, how many weeks will it take to save $.42?

$.06\overline{).42}$



At the bank, pennies are put into rolls, with $.50 in each roll. How many rolls can be made with $2.50 worth of pennies?

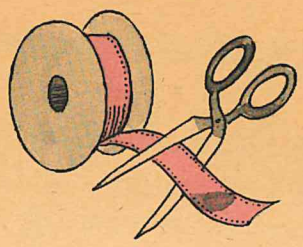
$.50\overline{)2.50}$ $\qquad$ $5\ 0\overline{)2\ 5\ 0}$



A bus ride costs $.15. How many rides can you take for $.90?

A boy is paid $.75 an hour for his work. How many hours does he have to work to earn $3.00?

A Mixed Number Divided by a Fraction or Mixed Number



How many $4\frac{1}{2}$ in. lengths of ribbon can be cut from a piece $13\frac{1}{2}$ in. long?

You must find $13\frac{1}{2} \div 4\frac{1}{2}$. First change the mixed numbers to improper fractions. Then use the method of page 89.

$13\frac{1}{2} \div 4\frac{1}{2} = \frac{27}{2} \div \frac{9}{2} = \frac{27}{2} \times \frac{2}{9} = \frac{54}{18} = \square$.

Divide.

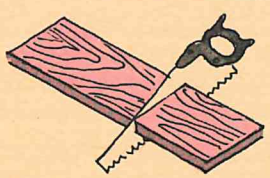
$2\frac{1}{2} \div \frac{1}{2} =$

$3\frac{1}{3} \div \frac{2}{3} =$

$2\frac{1}{2} \div 1\frac{1}{4} =$

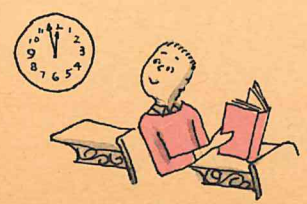
$4 \div 1\frac{1}{3} =$

$4\frac{1}{2} \div \frac{3}{4} =$



If a board is $10\frac{1}{2}$ ft. long, how many $3\frac{1}{2}$ ft. lengths can be cut from it?

How many people can be served with $31\frac{1}{2}$ oz. of tomato juice, if each serving is $4\frac{1}{2}$ oz.?

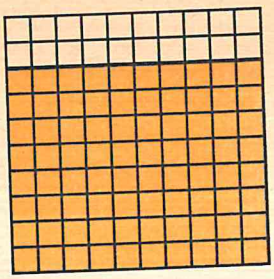


A school day is divided into periods that are $\frac{3}{4}$ hr. long. How many periods are there in a 3-hr. session?

Meaning of Per Cent

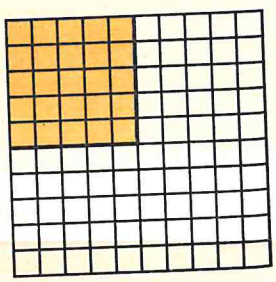
The symbol % is read as **per cent.** It means per hundred, or hundredths.

20% is read as 20 per cent. It means 20 hundredths, or $\frac{20}{100}$.



This large square is made up of 100 small squares. 20 out of 100 squares are white. $\frac{20}{100}$ of the squares are white. 20% of the squares are white.

80 out of 100 squares are in color. $\frac{80}{100}$ of the squares are in color. 80% of the squares are in color.



This large square contains 100 small squares.

☐☐ squares are in color.

$\frac{☐☐}{100}$ of the squares are in color.

☐☐ % of the squares are in color.

☐☐ % of the squares are white.

Complete these statements:

15 out of 100 = $\frac{15}{100}$ = ☐ %.

25 out of 100 = $\frac{☐☐}{100}$ = ☐ %.

40 out of 100 = $\frac{☐☐}{100}$ = ☐ %

30% means $\frac{30}{100}$ or 30 out of 100.

50% means $\frac{☐☐}{100}$ or ☐ out of 100.

75% means $\frac{☐☐}{100}$ or ☐ out of 100.

90% means $\frac{90}{☐☐}$ or 90 out of ☐.

60% means $\frac{60}{☐☐}$ or 60 out of ☐.

100% means $\frac{100}{☐☐}$ or 100 out of ☐.

Per Cent and Decimal Fraction

You now have three ways of writing hundredths: as a per cent, as a common fraction, and as a decimal fraction.

Example: 50% = $\frac{50}{100}$ = .50.

Write each number of hundredths in 3 ways.

30 hundredths = ☐☐ % = $\frac{☐☐}{100}$ = .☐☐.

65 hundredths = ☐☐ % = $\frac{☐☐}{100}$ = .☐☐.

90 hundredths = ☐☐ % = $\frac{☐☐}{100}$ = .☐☐.

5 hundredths = ☐☐ % = $\frac{☐☐}{100}$ = .☐☐.

Change to decimal fractions.

25% =	5% =	10% =
48% =	4% =	20% =
56% =	3% =	70% =

Write each of the following as a per cent.

.07 = 7%.	.42 =	.11 =
.09 =	.80 =	.10 =
.12 =	.75 =	.40 =

A test that Henry took had 100 questions. Henry got 85 answers right and the rest wrong.

☐☐ % of his answers were right.

Mr. Green bought a sheet of 100 postage stamps. He used 40 of the stamps.

☐☐ % of the stamps were used.

☐☐ % of the stamps were left.

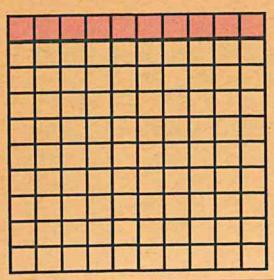
A dollar is worth ☐ cents.

A dime is ☐ % of a dollar.

A quarter is ☐ % of a dollar.

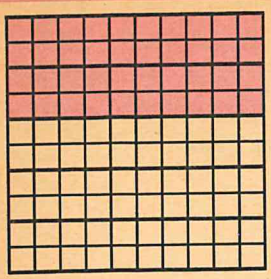
A half-dollar is ☐ % of a dollar.

Changing a Per Cent to a Fraction



10% of these squares are in color. Change this per cent to a common fraction reduced to lowest terms.

$10\% = \frac{10}{100} = \frac{1}{10}$.

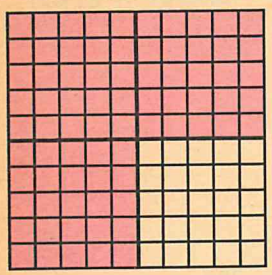


What per cent of these squares are in color?

Answer: ☐%

Change the per cent to a common fraction, and reduce.

☐% = $\frac{\square}{100} = \frac{\square}{\square}$.

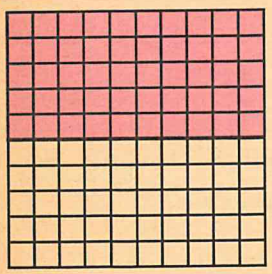


What per cent of these squares are in color?

Answer: ☐%

Change the per cent to a common fraction, and reduce.

☐% = $\frac{\square}{100} = \frac{\square}{\square}$.



What per cent of these squares are in color?

Answer: ☐%

Change the per cent to a common fraction, and reduce.

☐% = $\frac{\square}{100} = \frac{\square}{\square}$.

Change to fractions in lowest terms.

25% = $\frac{\square}{100} = \frac{\square}{\square}$.

5% = $\frac{\square}{100} = \frac{\square}{\square}$.

30% = $\frac{\square}{100} = \frac{\square}{\square}$.

15% = $\frac{\square}{100} = \frac{\square}{\square}$.

70% = $\frac{\square}{\square} = \frac{\square}{\square}$.

35% = $\frac{\square}{\square} = \frac{\square}{\square}$.

90% = $\frac{\square}{\square} = \frac{\square}{\square}$.

45% = $\frac{\square}{\square} = \frac{\square}{\square}$.

80% = $\frac{\square}{\square} = \frac{\square}{\square}$.

55% = $\frac{\square}{\square} = \frac{\square}{\square}$.

Expressing Parts as Per Cents

There are 20 questions on an arithmetic test. Alice answered 16 of the questions correctly. What per cent of her answers were correct?

$\frac{16}{20}$ of Alice's answers were correct. Change the fraction to hundredths, and then to a per cent.

$\frac{16}{20} = \frac{16 \times 5}{20 \times 5} = \frac{80}{100} = 80\%$.

Change each fraction to a per cent.

$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{\square}{100} = \square\%$.

$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{\square}{100} = \square\%$.

$\frac{1}{2} = \frac{1 \times \square}{2 \times \square} = \frac{\square}{100} = \square\%$.

$\frac{1}{10} = \frac{1 \times \square}{10 \times \square} = \frac{\square}{100} = \square\%$.

$\frac{3}{10} = \frac{\square}{100} = \square\%$.

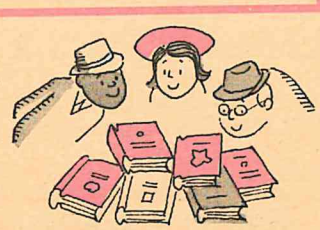
$\frac{7}{10} = \frac{\square}{\square} = \square\%$.

$\frac{9}{10} = \frac{\square}{\square} = \square\%$.

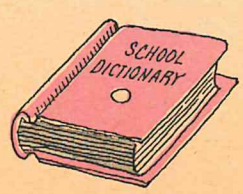
$\frac{1}{5} = \frac{\square}{100} = \square\%$.

$\frac{3}{5} = \frac{\square}{\square} = \square\%$.

$\frac{4}{5} = \frac{\square}{\square} = \square\%$.



During a book sale, the price of a $4 book was reduced by $1. The price was reduced by $1 out of $4 or ☐%.



If you spell 45 out of 50 words right, what is your spelling score?

$\frac{45}{50} = \frac{\square}{100} = \square\%$.

In the United States, 9 out of 10 children aged 14 to 17 are enrolled in school. What per cent of the children aged 14 to 17 are in school?

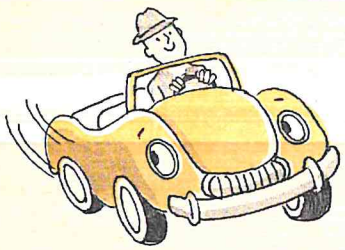
$\frac{9}{10} = \frac{\square}{100} = \square\%$.

Making Sense with Decimals

In the statements below, the decimal point has been left out of some of the numbers. Put the decimal point where it should be for each statement to make sense.

The patient's temperature in the morning was 1008 degrees.

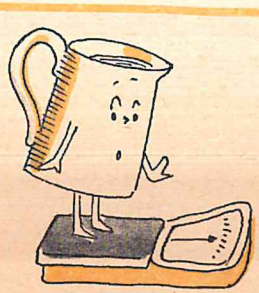
In 1956, David Sime set a record when he ran 220 yards in 200 seconds.



On the average, Mr. Brown's car travels 182 miles on 1 gallon of gasoline.

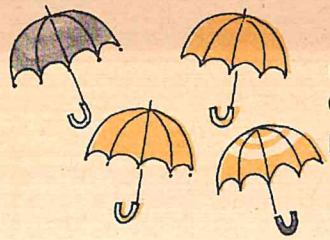
Sam traveled 40 miles on his bicycle. The trip took him 425 hours.

Mary works as a babysitter on weekends. She is paid $75 an hour.



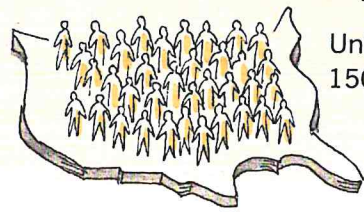
A quart of water weighs 22 pounds.

Putting in the Missing Decimal Point

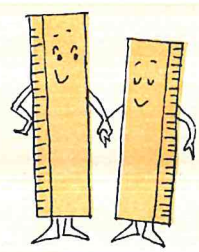


The average amount of rainfall in San Francisco, California, is 2051 inches per year.

On January 20, 1954, Rogers Pass, Minnesota, had a temperature of 697 degrees below zero.

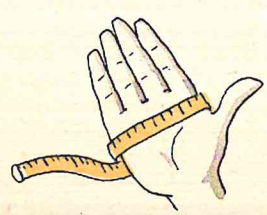


The population of the United States in 1950 was 1507 million.

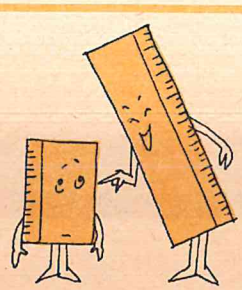


A meter stick is a little longer than a yard. It is 3937 inches long.

The baby weighed 75 pounds at birth.



The young man measured the palm of his hand and found that it was 325 inches wide.



A centimeter is a little less than half an inch long. There are 254 centimeters in an inch.

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by Irving Adler, Ph.D.

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What numbers mean • How numbers are written • What order numbers come in (or how to count) • How to add • How to subtract • How to compare sizes (or what **tall, small, more,** and **fewer** mean) • What **pennies, nickels,** and **dimes** are (or how to use pennies, nickels, and dimes) • How to make change • How to tell the hours on a clock • What a **foot ruler** is • What **triangles, squares,** and **circles** are • What order the days of the week come in • What a **dozen** is.

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MATHEMATICS—GRADE 5

What fractions are (what fraction means) • How to work with fractions • More about adding and subtracting • More about multiplying and dividing • More about Roman numerals • More about money • What **average** means • More about clocks and calendars (more about time) • More about weight • More about length.

MATHEMATICS—GRADE 6

How to count up to 100,000,000,000 • More about adding and subtracting • More about multiplying and dividing • More about fractions • What **mixed numbers** are • What **decimals** are • How to use decimals • How to work with **rate, time,** and **distance** • How to find **area** • How to find **perimeter** • How to find **volume** • More about Roman numerals • Some laws of arithmetic.

Name _____

IN THIS WORKBOOK, I WILL LEARN:

- how to count up to 100,000,000,000
- more about adding and subtracting
- more about multiplying and dividing
- more about fractions
- what mixed numbers are
- what decimals are
- how to use decimals
- how to work with rate, time, and distance
- how to find area
- how to find perimeter
- how to find volume
- more about Roman numerals
- some laws of arithmetic

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